Precision of Ratings*

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Abstract

We analyze the equilibrium precision of ratings. Our results suggest that ratings become less precise as the share of uninformed investors and the aggregate value of liquidity increase. The results provide an explanation for low accuracy of ABS ratings before the financial crisis. We apply the model to evaluate the effectiveness of the recent reform proposals, including Dodd-Frank Act. We show that some policies, in particular, rating standardization and expert liability, reduce market efficiency.

JEL codes: D82, D83, G01, G18, G24, G28, L15.

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1 Introduction

Credit rating agencies (CRAs) rate securities in various asset classes. The US Securities and Exchange Commission identifies five classes of ratings, (1) financial institutions, brokers and dealers; (2) insurance companies; (3) corporate issuers; (4) issuers of asset-backed securities; and (5) issuers of government, municipal or sovereign securities. These asset classes differ in terms of information asymmetries between the issuers and investors. For example, the investors assessment of the credit quality of sovereign securities can be based on publicly available sources summarizing countries macroeconomic conditions. To the contrary, investors may need special expertise to assess the credit quality of a mid -sized industrial company. When the information asymmetry is substantial, investors can be differentially informed about the assets quality. Also asset classes can differ in terms of the

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issuer's value of liquidity and the availability of positive NPV investment opportunities in a particular class.

The performance of CRAs during the financial crises suggests that the precision of ratings varies across asset classes. Several empirical studies report that the ratings of asset-backed securities were uninformative and inflated to highest AAA rating.[1] In August 2011, the US Justice Department started an investigation whether one of the major CRAs, Standard and Poor's (S&P), improperly rated mortgage securities in the year prior to the financial crisis. At the same time, the performance of ratings in corporate bond market, utilities and insurance sectors was stable, even during the times of the financial crisis.[2]

The purpose of the paper is to explain what determines the precision of ratings. We argue that the incentives of the CRA to produce accurate ratings depend on the market conditions measured by the aggregate value of liquidity, the distribution of assets in the economy and the extend of the winner's curse problem among the heterogeneously informed investors. We build a rational model that incorporates these factors and apply it to analyze the effect of the recent CRAs reforms proposals, in particular, the Dodd-Frank Act, on the equilibrium precision of ratings.

We model a market with issuers, investors and a monopolistic CRA. Issuers are privately informed about the value of the asset and aim to sell the issue at the highest price. Issuers need a rating to signal the asset quality to investors. The CRA designs the rating system that is composed of the information technology and a rating fee. This set up follows the information intermediation literature (Lizzeri 1999).

We introduce two novel features to the information intermediation literature, the issuers' value of liquidity and the presence of differentially informed investors. We assume that issuer's outside option is increasing in the asset quality and proportional to the aggregate value of liquidity. When the issuers have a lot of attractive investment opportunities, they are willing to accept a higher discount to sell the issue. As we show, the value of

[1] Ashcraft, Goldsmith-Pinkham, and Vickery find that subprime and Alt-A mortgage backed securities (MBS) experienced a significant decline in rating standards, and 80-95% of deals were assigned a AAA rating. Stanton and Wallace (2011) document that ratings of commercial mortgage-backed securities allowed for lower subordination levels that inflated the ratings. The size of the AAA tranche of collateralized debt obligation (CDO) deals was larger than suggested by the CRA's rating model (Griffin and Tang, 2009), with low B+ credit quality of the collateral that supported CDO issues (Benmelech and Dlugosz, 2009).

[2] According to Standard and Poor's report on corporate default rates and rating transitions, during 2008-09 only 25 companies initially rated as investment grade were in default, and the number of investment grade defaults was at most one per year during the rest of the period of 2004-2011. The rate of speculative grade companies defaults peaked to 9.5% in 2009, which is comparable to 9.7% rate following the high-tech bubble in 2001.

liquidity will have an important negative feedback effect on the precision of ratings.

The second novel feature is that we consider a market with differentially informed investors. In this setting, the CRA plays an active role in creating the market surplus. Increasing the precision of ratings limits the ability of informed investors capture the information rent, and thus enlarges the market surplus by solving the winner's curse problem. However, we show that more precise ratings also reduce the ability of the CRA to extract the surplus, and the optimal information structure results as a trade-off of the two countervailing incentives.

The value of liquidity and the presence of differentially informed investors is what distinguishes asset classes and varies through time. Thus the model can explain the heterogeneous performance of CRAs. Also it provides a general framework that can be applied to evaluate a variety of the recent proposals on the reforms of CRAs in the US and in Europe. These proposals include the reforms on standardization of ratings across different asset classes, regulation of rating fees, introducing expert liability for overrated securities and reducing the reliance on ratings in regulation.

We obtain five main results. The first result is that the CRA's ratings are informative but noisy. It is driven by the fact that the profit of the CRA is a product of market penetration and the fee. In the extreme case when ratings are perfectly informative about asset values, the rating fee is determined by the willingness to pay of the lowest rated issuer. The CRA can increase this issuer's willingness to pay by assigning it high ratings with a positive probability. However, in doing so, the CRA is limited by the outside option of high quality issuers. If ratings are uninformative, high quality issuers prefer to hold the asset instead of selling it at a substantial discount. This result contrasts with Lizzeri (1999) where the ratings are completely uninformative. The trade off between increasing the willingness to pay of lower quality issuers and revealing enough information to induce participation of high quality issuers determines the precision of ratings.

The second result is that precision of ratings depends on the market conditions. When the value of liquidity is high, issuers are willing to accept a higher discount to sell the asset. It gives the CRA a possibility to extract more surplus by making ratings less informative. Thus as the aggregate value of liquidity increases, the precision of ratings is compromised.

The third result is about the precision of ratings in the market with differentially informed investors. When all investors are uninformed, the CRA's information structure affects the distribution of surplus between the issuers and the CRA, but it does not affect the size of surplus. In the presence of the winner's curse problem, the information structure of the CRA changes the size of surplus as informed investors capture the information rent. The reason is that more informative ratings reduce the adverse selection problem of uninformed investors and increase the surplus. At the same time, more informative ratings reduce the ability of the CRA to extract issuers surplus. We show that as the share of uninformed investors increases, the CRA reduces the precision of ratings. Also when the winner's curse problem is substantial, the CRA reduces the market coverage to the best quality issuers.

Fourth, we provide several results about the optimal information structure of the CRA. We show that the information structure is asymmetric and, under certain conditions, must entail rating inflation. That is, lower quality issuers must be assigned higher ratings with a positive probability, but higher quality issuers are always assigned high ratings. Otherwise, higher quality issuers may refuse to trade following a low rating, which reduces CRAs profits. Thus the CRA guarantees that its "mistake" is always optimistic.

Finally, we show that the precision of ratings depends on the distribution of asset values in the economy. As the high quality assets become more scares, the precision of ratings decreases. The reason is that rating lower quality assets becomes a more important source of CRAs profits, and these issuers willingness to pay is increasing as ratings become less informative.

We apply the model to evaluate the recent reform proposals of the credit rating industry. Following disappointing performance of ratings of asset backed securities, regulators both in the US and in the EU developed an array of policies to improve incentives of CRAs to produce accurate ratings. We discuss the Dodd-Frank proposals on ratings standardization, introducing expert liability and reliance on ratings in regulation, as well as several proposals regarding regulation of rating fees. Our analysis suggests that some of these policies can be detrimental to welfare. In particular, we show that standardization of ratings across asset classes and introducing expert liability can reduce the precision of ratings and market liquidity. To the contrary, reducing reliance on ratings in regulation and regulating rating fees can increase welfare.

The rest of the paper is organized as follows. The next section reviews the related literature. Section 3 describes the model. Section 4 derives several properties of the information structure, and Section 5 applies the results to characterize the optimal precision of ratings. Evaluation of policy proposals on the CRA reform is contained in Section 6, and the conclusion follows. All proofs are devoted to the Appendix.

2 Related literature

We build on a framework developed in Lizzeri (1999) that derives several important results in the information intermediation literature. Lizzeri considers a model with a continuum of seller types, risk neutral buyers and no restrictions on the disclosure rules that the information intermediary can offer to sellers willing to pay for certification. He shows that the information intermediary (CRA) captures all the surplus and discloses no information.

The basic logic that drives the outcome is as follows. The profit of an intermediary is a product of the market penetration and the fee charged for the certification services. In a market where buyers' certification decision is voluntary, a fee that an intermediary can charge is determined by the willingness to pay of the lowest rated seller. If an intermediary discloses the type of the lowest rated seller perfectly, the seller is willing to pay at most the difference between its type and the average value of uncertified sellers with lower types. However, without decreasing the market penetration, an intermediary can increase the willingness to pay of the lowest rated type by pooling it with higher types. As higher types have no means to signal their quality other than the intermediary's certification, the optimal disclosure of an intermediary is to pool all types. It implies that ex-post the intermediary discloses no information. Also it is able to extract all the surplus by charging the fee equal to the expected value in the market.

We depart from Lizzeri's basic model in that the issuer (the seller) has an outside option that is increasing in its type. Also we introduce the adverse selection problem by assuming that investors are heterogeneously informed.

Several theories have been proposed to explain the low performance of CRAs during the financial crisis. Mathis, McAndrews and Rochet (2009) show that reputation is sufficient to discipline CRAs only when a large fraction of their incomes come from rating simple assets. Benabou and Laroque (1992) analyze the incentives of an intermediary to manipulate information when the intermediary also acts as a speculator on the market. Goel and Thakor (2010) show that CRAs may have incentives to produce coarse ratings in order to reduce the litigation risk.

The complexity of information and participation of naive buyers can lead to biases in information reporting. Skreta and Veldkamp (2008) study how the higher complexity of rated assets affects incentives for ratings shopping. They show that the ability of sellers to compare ratings from different CRAs before the ratings are disclosed to the market leads to ratings shopping and ultimately inflates ratings. Bolton, Freixas and Shapiro (2012) show that a CRA may overstate the seller's quality when there are more naive investors.

Opp, Opp and Harris (2011) show that regulation can induce ratings inflation.

We contribute to the literature by developing a general rational framework that incorporates many important features of the market and allows to analyze the effect of changing market conditions on the precision of ratings. Also we apply our framework to analyze the array of policy proposals on CRA reform within a scope of a single model.

We analyze a general information structure of the CRA. In this respect our paper is related to mechanism design literature on optimal information structures in auctions, Bergemann and Pesendorfer (2007).

3 Model

There are three groups of agents: issuers, investors and a CRA. An issuer owns an asset that is worth v to investors and δv to an issuer. The issuers have liquidity needs and value the asset less than the investors, $\delta < 1$. As δ decreases, the liquidity needs of the issuer increase. The CRA does not trade the asset. Issuers are privately informed about v, and the investors and the CRA have a prior on the value of v represented by the distribution $\lambda = (\lambda_1, \lambda_2, \lambda_3)$, $\lambda_i = \Pr(v_i) > 0$, $\Sigma_i \lambda_i = 1$ on the support $V = \{v_1, v_2, v_3\}$, $0 = v_1 < v_2 < v_3$. Issuers' types are independent.

Investors consist of two groups, informed and uninformed. Uninformed investors are purely competitive and represent a group large enough to buy the entire issue. Uninformed investors know that there is a possibility that there are some informed investors who observe the value of the asset v prior to subscribing to the issue. Demand of informed investors is not sufficient to absorb the entire issue. All investors demand a fixed amount of the issue as long as the expected value is higher than the price. Uninformed investors face a winner's curse problem. They are more likely to obtain a larger allotment when informed investors decide not to subscribe to an issue. The rationing rule between the two groups of investors is summarized by the probability q that the uninformed investor's demand for an underpriced security is fulfilled. Furthermore, the uninformed investors' demand is fulfilled with probability one if only uninformed investors demand. The probability q measures the severity of the winner's curse problem. If all investors are uninformed and there is no winner's curse, then $q = 1$. As q decreases, the share of informed investors increases. This approach builds on Rock (1986).

The CRA has an information technology to evaluate the value of the asset. The signal space is denoted by $S = \{s_1, ..., s_M\}$, $M \leq +\infty$. An information structure I is given by a pair $\langle S, F(v, s)\rangle$, where $F(v, s)$ is the joint probability distribution over the set of asset values V and the set of signals S. The joint probability distribution is defined in a usual way,

$$F(v,s) = \Pr(\widetilde{v} \leq v, \widetilde{s} \leq s),$$

with $f(v_j, s_i) = \Pr(v = v_j, s = s_i)$. For F to be part of the information structure requires that the marginal distribution with respect to v to be equal to the prior distribution of v, $\sum_i f(v_j, s_i) = \lambda_j$. Let $\mathcal{I}$ denote the set of information structures that satisfy this condition. For a given set of signals S, the precision of a signal s_i on type v_j is defined by

$$p_{ij} = \Pr(s_i|v_j) = \frac{f(v_j, s_i)}{\Sigma_i f(v_j, s_i)}. \tag{1}$$

The CRA can choose any information structure. The cost of every information structure to the CRA is equal to zero. CRA charges a flat fee $\phi \geq 0$ to an issuer soliciting a rating and commits to reveal the signal realization to investors. The profile (I, ϕ) defines the rating technology of the CRA. The choice of the rating technology is common knowledge among issuers and investors.

The information structure permits a very rich set of rating systems. The information structure yields perfect information if $M \geq 3$ and $p_{ij} = 1$ when $i = j$ and $p_{ij} = 0$ otherwise. The information structure is uninformative if for some $s_i \in S$, $p_{ij} = 1$ for all j, and $p_{ij} = 0$ otherwise. A rating system with rating grades can be represented with an information structure where a subset of type $V_i \subset V$ is assigned the same signal s_i, $p_{ij} = 1$ for all $v_j \in V_i$ and $p_{ij} = 0$ otherwise. A noisy rating system is a system where the same type can be assigned different signals, $p_{ij} < 1$ for all i, j.

The structure of the game is common knowledge to issuers, informed and uninformed investors and the CRA. The timing of the game is as follows.

$t = 0$. The nature chooses the issuer's type according to the prior distribution λ. Issuers privately learn their types $v \in V$. The rating agency commits to the rating technology (I, ϕ). The rating technology (I, ϕ) is observed by the issuers and the investors.

$t = 1$. Issuers decide whether to pay the fee and solicit a rating from the CRA. Informed investors learn the value of the asset for each issuer v and the CRA learns a signal s for issuers who solicited a rating. Rating agency announces the ratings of rated issuers.

$t = 2$. Issuers set the price of subscription b.

$t = 3$. Investors who have observed whether the issuer is rated and the assigned rating at $t = 1$, decide whether to subscribe to an issue. The demand of informed and uninformed investors is fulfilled according the rationing rule summarized above.

The strategy for the CRA is the information structure I and a fee ϕ. A behavioral strategy for the issuer is a pair of functions $d : V \times \mathcal{I} \times \mathcal{R}_+ \to [0, 1]$ that maps the issuer's type v and the rating technology (I, ϕ) into the probability to solicit a rating d, and $b : V \times \mathcal{I} \times \mathcal{R}_+ \times S \to \mathcal{R}_+$ that maps the issuer's type v, the rating technology (I, ϕ) and the realization of the signal s into the price of subscription b. A strategy of the investor is a decision to subscribe to an issue given the information available at $t = 3$. For informed investors, the subscription decision is a function $\beta_I : V \times \mathcal{R}_+ \to \{0, 1\}$ that maps the issuer's type v and the price of subscription b into the decision to subscribe (1) or not (0). For uninformed investors, the subscription decision is a function β_U : $\mathcal{I} \times \mathcal{R}_+ \times \{0, 1\} \times S \times \mathcal{R}_+ \to \{0, 1\}$ that maps the rating technology (I, ϕ), the decision of an issuer to get rated (1) or not (0), the realization of the signal s and the price of the subscription b into the decision to subscribe (1) or not (0). We analyze the set of Perfect Bayesian equilibria of the game.

The model shares the basic framework of Lizzeri (1999). It departs from Lizzeri's model in two important dimensions. The first difference is related to the value of the asset to issuers (sellers). In Lizzeri's model, the issuer's value for the asset is equal to zero for all issuer types. Assuming that the issuers' outside opportunity δv is proportional to their types allows to capture an important feature of the financial market that higher quality issuers have more flexibility in raising external financing. The common discount δ measures the aggregate value of liquidity. In the context of Lizzeri's model, $\delta = 0$.

The second difference is related to the information available to investors (buyers). Lizzeri assumes that all investors are uninformed. This assumption implies that the total surplus does not depend on the information produced by the CRA. As we will show below, in the market with differentially informed investors the CRA's choice of the information structure affects the size of the surplus. The amount of information available to investors is captured by the probability q that uninformed investors' demand for underpriced issue is fulfilled. As q gets arbitrary small, the fraction of uninformed investors goes to zero. In the Lizzeri's model, all investors are uninformed and $q = 1$.

There are two technical differences between our model and Lizzeri (1999) that do not affect the qualitative results but make the model tractable. Lizzeri considers a continuum of types v on a bounded interval while we restrict attention to discrete finite types. The restriction permits the equilibrium analysis of the market with differentially informed investors. Also we model the information technology of the CRA as an information structure while Lizzeri considers a general set of disclosure policies. Given that no cost is imposed on the choice of the information structure or the disclosure policy, the two approaches are equivalent.

For a given information structure I, consider the decision of investors to subscribe to an issue at time $t = 3$. Let $\gamma_{ij} = \Pr(v_j | I, s_i)$ denote the beliefs of uninformed investors that an issuer rated $s_i \in S$ under the rating system I is type v_j, conditional on an issue offer. Also denote s_0 the event that an issuer is not rated, and $\gamma_{0j} = \Pr(v_j | I, s_0)$ the corresponding beliefs of uninformed investors conditional on the issue offer. The uninformed investors' assessment of the asset value of an issuer rated s_i, $i = 0, 1, ..., M$ under rating system I is

$$U_i = \sum_j \gamma_{ij} v_j.$$

They decide to subscribe to an issue rated s_i if the price of subscription b_i does not exceed their assessment of the asset value,

$$b_i \leq U_i.$$

At time $t = 2$, the issuer type v_j is better off selling the issue rated s_i as long as the price is higher than the issuer's asset value,

$$b_i \geq \delta v_j.$$

It means that there are gains from trade for issuer type v_j when

$$U_i \geq \delta v_j. \tag{2}$$

Otherwise, the issuer holds the asset, for example, by setting the subscription price equal to $\overline{v}$, where $\overline{v} > v_3$.

Condition (2) defines the set of issuers $T_i \subset V$ that are willing to trade if obtain a rating s_i under the rating system I,

$$T_i = \{v_j | U_i \geq \delta v_j\}.$$

The issuers T_i optimally set the price

$$b_i = U_i.$$

At this price, the uninformed investors break even. If the issue is underpriced, informed investors gain a positive rent equal to the difference between the asset value and the price, $v_j - U_i$.

At time $t = 1$, if an issuer type v_j solicits a rating, it is assigned a rating s_i with probability p_{ij}. Given the rating, at stage $t = 2$ the issuer can either charge the price U_i

or hold the asset and realize the value δv_j. Thus issuer's expected payoff if it solicits a rating is

$$R_j = \sum_i p_{ij} \max\{U_i, \delta v_j\}.$$

If an issuer does not solicit a rating, it can either sell the issue unrated at price U_0 or hold the asset. Then the payoff of unrated issuer is equal to $\max\{U_0, \delta v_j\}$. Given a rating technology (I, ϕ), denote $d_j \in \{0, 1\}$ the decision of type v_j to solicit a rating,

$$d_j = \begin{cases} 1 \text{ if } R_j - \phi - \max\{U_0, \delta v_j\} \geq 0, \\ 0 \text{ otherwise.} \end{cases}$$

At stage $t = 0$, the CRA chooses a rating technology (I, ϕ) that maximize its expected profit,

$$\Pi(I, \phi) = \sum_j \lambda_j d_j \phi.$$

In our model, the CRA may be necessary to realize the gains of trade. If there is no information intermediary and all issuers' types sell the asset, the expected value of the asset under the prior distribution λ is $E[v] = \lambda_2 v_2 + \lambda_3 v_3$. However, when the outside option of type v_3 is high, $\delta v_3 > E[v]$, the issuer v_3 prefers to hold the asset. If only issuer types v_1 and v_2 trade, the expected value of the asset is $\frac{\lambda_2 v_2}{\lambda_1 + \lambda_2}$. But again, high value of outside option, $\delta v_2 > \frac{\lambda_2 v_2}{\lambda_1 + \lambda_2}$, can induce type v_2 to hold the asset. If both inequalities hold, there is no trade and the outcome is inefficient. This is the usual Akerlof's (1970) market for lemons problem. Note that if the issuer does not have an outside option, $\delta = 0$, the CRA is not needed as all issuers can trade at price $E[v]$.

Now suppose that the CRA rates all issuers and ratings are perfectly informative about issuers types. Then each issuer type v_i sells at price v_i and realizes the surplus of $(1 - \delta)v_i$. Thus the CRA can restore the market efficiency and solve the lemons problem. However, we show that the CRA has different incentives. In the next section we develop the properties of the information structure that maximizes CRA's payoff.

4 Characterization of the information structure

In this section, we derive several properties of the CRA's optimal information structure. We first present the result that an optimal information structure equates the rated issuers' willingness to pay for CRA's services. Then we analyze how the extend of asymmetric information among investors affects the set of rated issuers, the ability of the CRA to extract the market surplus and the precision of an optimal information structure.

Consider the set of issuers that solicit a rating of the CRA under a given rating system (I, ϕ). The minimum payoff that a rated issuer type v_j must receive net of the rating fee is its outside option δv_j. Otherwise, the issuer is better off without a rating. The CRA extracts the surplus of rated issues if type v_j issuer's expected payoff of soliciting a rating R_j is such that $R_j = \delta v_j + \phi$. Then an optimal rating system must have the following properties.

Proposition 1 *Under an optimal rating system, it must be that*

$$R_i - \delta v_i = R_j - \delta v_j = \phi$$

for all rated types $i, j \in \{h | d_h = 1\}$. *The willingness to pay for the rating is increasing in issuer's type,* $R_H \geq R_M \geq R_L$.

The economic intuition of these properties is as follows. Due to the flexibility of the information structure, the CRA can continuously change the willingness to pay for the rating. If the net value of a rating differs for any two rated types, $R_i - \delta v_i > R_j - \delta v_j$, the fee is equal to the willingness to pay of the type with the lowest net valuation, $\phi = R_j - \delta v_j$. Then, without changing the set of rated types, the CRA can charge a higher fee by increasing type v_j willingness to pay for the rating. Thus the optimal rating system must equate all types willingness to pay. The monotonicity property follows immediately from this result and that issuers' outside opportunity is increasing in their type.

In the subsequent analysis, we distinguish between the situations of uninformed, $q = 1$, and heterogeneously informed, $q < 1$, investors. The reason is that the extend of asymmetric information among investors affects the impact of the CRA on the market surplus. When all investors are uninformed, the market surplus is fixed. Then the CRA designs the information structure that aims to maximize its share of the surplus. We show that there exist an information structure that permits full surplus extraction. In the market with differentially informed investors, the CRA plays an active role in creating the surplus. The reason is that increasing the precision of the information structure reduces the adverse selection problem, and thus increases the market surplus between the issuers and the CRA. However, as we show below, higher precision also reduces the ability of the CRA to extract the surplus. Thus the optimal information structure will trade off these two countervailing incentives, leading to partial extraction of market surplus. Furthermore, we analyze how the winner's curse problem affects the set of issuers' types that solicit a rating.

4.1 Investors with no private information

In the market with uninformed investors, issuers' do not face a winner's curse problem and the market surplus is fixed. The size of the surplus is equal to the expected value of trade, $(1-\delta)E[v]$. The main result of this section is that CRA can design the optimal information structure to fully extract the surplus.

Proposition 2 *When all investors are uninformed, $q = 1$, the CRA fully extracts the market surplus and it is indifferent between rating all types or rating two top types v_2 and v_3. These strategies dominate rating only one type v_3.*

Issuers willingness to pay for the rating depends on the CRA's market penetration. As the CRA's rating system excludes participation of lower types, rated types' willingness to pay increases. When k top types are rated, type v_j willingness to pay, R_j^k, equals

$$R_j^3 = (1-\delta)E[v] + \delta v_j,\ j = 1, 2, 3,$$

$$R_j^2 = \frac{(1-\delta)E[v]}{\lambda_2 + \lambda_3} + \delta v_j,\ j = 2, 3,$$

$$R_3^1 = v_3.$$

The proposition implies that the optimal information structure is noisy. To ensure that the net value of a rating, $R_j^k - \delta v_j$, is constant across types, the CRA must increase the lower types willingness to pay by assigning the same signals to lower and higher types with a positive probability. It cannot be achieved under the fully revealing information structure that would lead to unequal net values $v_j - \delta v_j$.

The indifference between rating two or three types follows from the observation that the lowest type v_1 does not need a rating to realize its value. Then the CRA can distribute the ex-post valuations of rated issuers' among two or three types without affecting the amount of its surplus. However, further reduction of market coverage to rating only the highest type v_3 reduces CRA's payoff. The reason is that the strategy foregoes the surplus created when issues type v_2 differentiate themselves from the lowest type v_1.

The properties of Proposition 2 also hold in a general case with $n \in \mathbb{N}$ types of issuers. Indeed, note that the willingness to pay under full market coverage R_j^3 does not depend on that there are three issuers' types, and $R_j^n = R_j^3$. The CRA is indifferent between rating n and $n-1$ types for the same reasons as discussed earlier. Rating $n-1$ types leads to issuer willingness to pay

$$R_j^{n-1} = \frac{(1-\delta)E[v]}{\sum_{\iota=2}^{n} \lambda_i} + \delta v_j,\ j = 2, \ldots, n.$$

The next proposition describes how the issuers' willingness to pay depends on the value of liquidity $1-\delta$.

Proposition 3 *The precision of ratings increases as the value of liquidity* $1-\delta$ *declines.*

When the value of liquidity is very high, $\delta = 0$, the issuers gain no profit from holding the asset and are eager to sell the issue to realize the value, even at a discount. As in Lizzeri (1999), the CRA designs uninformative ratings that issuers solicit to distinguish themselves from the worse type. However, as the value of liquidity declines and δ increases, high quality issuers are better off holding the asset and realizing the value δv instead of selling it at substantial discount due to uninformative ratings. In order to maintain full market coverage, the CRA has to reveal enough information to induce participation of higher types. In the limit case $\delta = 1$, the CRA discloses all information.

The result of Proposition 3 also suggests that the ability of the CRA to gains profits must be procyclical. When the economy is in a boom, holding the assets is costly due to attractive investment opportuntities and δ is low. Then issuers have high opportunity costs of capital and are eager to sell the issue, yielding high profits to the CRA. When the economy is bust and δ is high, there are few investment opportunities, and the profits of the CRA are low.

Next, we show that CRA can achieve its goal by using only three signals. In order to illustrate the economic intuition, consider an example of perfectly revealing information structure where $S = \{s_1, s_2, s_3\}$ and $p_{jj} = 1$, $j = 1, 2, 3$, $p_{ij} = 0$ for all $i \neq j$, $i = 1, 2, 3$. Then the willingness to pay for the rating is $R_j = v_j$. Under full market coverage, CRA is constrained to charge zero fee for the services, $\phi = R_1 = v_1 = 0$, and it gains no profit. The CRA can increase the profit by reducing the market coverage to types v_2 and v_3, and charging the fee $\phi = R_2 - \delta v_2 = (1-\delta)v_2$. In this case, it obtains profits $(\lambda_2 + \lambda_3)(1-\delta)v_2$, and type v_3 issuers gain the rest of the surplus, $\lambda_3(1-\delta)(v_3 - v_2)$. The CRA can further increase its profits by implementing the information structure that distributes the surplus from type v_3 to type v_2, and extracting this surplus via a higher rating fee. It can be achieved when type v_2 and v_3 are assigned the same signal with a positive probability. Below, we characterize an equilibrium information structure that has these properties.

Proposition 4 *The CRA extracts the market surplus under an information structure that employs three signals $\{s_1, s_2, s_3\}$ with precisions*

	v_3	v_2	v_1
s_3	1	$1-p_{22}$	0
s_2	0	p_{22}	0
s_1	0	0	1

where $p_{22} = \frac{\delta(\lambda_3+\lambda_2)}{\lambda_3+\delta\lambda_2} < 1$, and the rating fee $\phi = (1-\delta)\frac{\lambda_3 v_3+\lambda_2 v_2}{\lambda_3+\lambda_2}$. Issuers types v_3 and v_2 solicit a rating, while issuer type v_1 does not solicit a rating. All rated issuers trade the asset. Rating precision, p_{22}, is strictly increasing in δ.

Signals s_1 and s_2 lead to perfect revelation of types v_1 and v_2, respectively. Signal s_3 is assigned to two types, v_2 and v_3, with probabilities p_{22} and $1-p_{22}$. It is natural to interpret p_{22} as rating precision since higher values of p_{22} correspond to more separation between types v_2 and v_3 conditional on the rating. Under imperfect precision $p_{22} < 1$, the CRA extracts surplus from type v_3 by inflating the ratings of type v_2, and thus preventing the highest quality issuers to perfectly separate from issuers type v_2. As a result, both rated types v_2 and v_3 have the same willingness to pay for the rating equal to the rating fee ϕ.

The optimal rating system described in Proposition 4 need not be unique. From the perspective of the CRA, the optimality of the information structure requires that the signals prevent perfect separation between types v_2 and v_3. Given the rich set of information structures considered in our model, the objective can be reached in multiple ways.

The information structure in Proposition 4 entails rating inflation in a sense that the intermediate type issuers are assigned high ratings with a positive probability. In line with the discussion above, it does not require that rating inflation property holds for all information structures. However, under certain conditions rating inflation is a necessary property of the equilibrium rating system.

Proposition 5 *If the issuers have high outside option $\delta > \overline{\delta} = \frac{\lambda_2 v_2+\lambda_3 v_3}{\lambda_2 v_3+\lambda_3(2v_3-v_2)}$, the equilibrium information structure must entail rating inflation.*

To see why rating inflation is necessary, consider the case where type v_3 also receives a noisy signal. Without loss of generality, assume that he receives either s_2 or s_3, each with a positive probability. However, if δ is sufficiently high then we may have $U_2 < \delta v_3$

which will lead to type v_3 to set a too high price effectively withdrawing from the market. This withdrawal is inefficient and CRA can profitably deviate to an information structure where the highest type never receives a signal other than the highest signal.

4.2 Differentially informed investors

Privately informed investors will be able to gain some informational rent. Furthermore, the issue price must attract both informed and uninformed investors for the successful placement. It implies that the issuers must offer a winner's curse discount to uninformed investors and sell the issue at a price below its expected value. The extend of underpricing depends on the accuracy of information produced by the CRA. As we analyzed in the previous section, when all investors are uninformed, the CRA chooses a noisy information structure. With privately informed investors, noisy ratings come at a cost of underpricing. In addition to the incentives to provide inaccurate information to capture the market surplus, the CRA also has the incentives to improve the precision of rating to reduce the extend of underpricing. This intuition leads to the next important result regarding the CRA's market coverage in the market with differentially informed investors.

Proposition 6 *In the market with differentially informed investors, an information structure that induces participation of the two highest types v_2 and v_3 is more profitable than the one that induces rating all types. Consequently, rating all types is never an equilibrium.*

The result suggests that in the market with differentially informed investors the CRA provides partial market coverage. The economic intuition of the result is that including the lowest issuer type v_1 hardens the underpricing problem without increasing the market surplus that the CRA can obtain from issuers. The last observation follows from Proposition 2 that states that in the absence of adverse selection the CRA is indifferent between rating two or three types. Thus in the market with differentially informed investors the CRA chooses to reduce the market coverage. As we will see in the next section, as the extend of winner's curse problem increases, the CRA may choose to decrease the market coverage even further and rate only the highest issuer type.

In the next section we derive the optimal precision of ratings in a general case of differentially informed investors. In the analysis, we build on Proposition 6 and restrict attention to information structures that discourage participation of the lowest type v_1. Furthermore, we consider information structures with three signals. As we show in Proposition 4, three signals are sufficient to achieve the first best in the market with uninformed investors. Using this information structure in the market with differentially informed investors permits to have a benchmark to evaluate the effect of asymmetric information on the market outcome. Also it allows us to focus on the main question of the paper, that is, the effect of the winner's curse problem and market conditions on precision of ratings.

5 Optimal precision of ratings

In this section we characterize the profit maximizing information structure of the CRA. Also we analyze how the precision of ratings depends on the market conditions. The CRA's rating system is described by a set of probabilities $\{p_{ij}\}$, $i, j = 1, 2, 3$, where $p_{ij} = \Pr(s_i | v_j)$ is the precision of signal s_i about type v_j,

	v_3	v_2	v_1
s_3	p_{33}	p_{32}	p_{31}
s_2	p_{23}	p_{22}	p_{21}
s_1	p_{13}	p_{12}	p_{11}

It satisfies $\Sigma_i p_{ij} = 1$ for all $j = 1, 2, 3$.

The results of Proposition 6 suggest that in the market with differentially informed investors the CRA will never offer a rating system that induces solicitation of ratings by all types of issuers. Thus one can restrict attention to two equilibria in which the CRA targets either the highest type v_3 or the two highest types v_2 and v_3. The next proposition characterizes an optimal information structure in the former case.

Proposition 7 *If the target market coverage are issuers type v_3, the CRA sets the information structure*

	v_3	v_2	v_1
s_3	1	0	0
s_2	0	p_{22}	p_{21}
s_1	0	p_{12}	p_{11}

with $p_{ij} \in [0, 1]$ and $\Sigma_i p_{ij} = 1$ for $i, j = 1, 2$. It charges the fee $\phi = \min\{(1-\delta)v_3, v_3 - \frac{\lambda_2 q v_2}{\lambda_2+\lambda_1}\}$ and gains profit $\Pi_3 = \lambda_3 \min\{(1-\delta)v_3, v_3 - \frac{\lambda_2 q v_2}{\lambda_2+\lambda_1}\}$. Issuers type v_3 solicit a rating while issuers type v_2 and v_1 are not rated. Issuers type v_3 sell the asset at price v_3. If $\delta < \frac{q\lambda_2}{\lambda_2+\lambda_1}$, issuers v_1 and v_2 sell the asset at price $\frac{\lambda_2 q v_2}{\lambda_2+\lambda_1}$; otherwise, they do not trade.

Under the limited market coverage of issuer types v_3, the rating perfectly reveals the issuer type. The issuer thus can set the price equal to the value of the asset. If the outside option δv_3 of a rated issuer v_3 is higher than its value of trade without a rating, $\frac{\lambda_2 q v_2}{\lambda_2 q+\lambda_1}$, the CRA can extract the value of trade $(1-\delta)v_3$; otherwise, the CRA charges the fee which is equal to the gains the issuer v_3 obtains by differentiating from the other two issuer types. The lower quality issuers do not solicit a rating under this rating system because the fee charged by the CRA is higher that the value that these issuers can realize if rated. They may still trade the asset if the issuer v_2 prefers selling the asset at price $\frac{\lambda_2 q v_2}{\lambda_2+\lambda_1}$ rather than holding it and gaining δv_2. It occurs when $\delta < \frac{q\lambda_2}{\lambda_2+\lambda_1}$.

Now suppose that the CRA aims to design a rating system that induces rating solicitation from types v_2 and v_3. Consider the following information structure.

	v_3	v_2	v_1
s_3	p_{33}	p_{32}	0
s_2	p_{23}	p_{22}	0
s_1	0	0	1

Rating s_1 reveals perfectly type v_1. For any positive rating fee, type v_1 will choose not to solicit a rating under this information structure. Formally, the uninformed investors offered a issue rates s_i hold the beliefs $\gamma_{ij} = \Pr(v_j|\, s_i)$, with

$$\begin{aligned}
\gamma_{33} &= \frac{\lambda_3 q p_{33}}{\lambda_3 q p_{33} + \lambda_2(1-p_{22})},\\
\gamma_{22} &= \frac{\lambda_2 p_{22}}{\lambda_3 q(1-p_{33}) + \lambda_2 p_{22}},
\end{aligned}$$

$$\gamma_{32} = 1-\gamma_{33} \text{ and } \gamma_{23} = 1-\gamma_{22},$$

$$\gamma_{31} = \gamma_{31} = \gamma_{13} = \gamma_{12} = 0, \gamma_{11} = 1$$

The resulting uninformed investors' assessment of the assets rated s_3 and s_2 are

$$U_3 = \gamma_{33}v_3 + \gamma_{32}v_2 = \frac{\lambda_3 q p_{33} v_3 + \lambda_2(1-p_{22})v_2}{\lambda_3 q p_{33} + \lambda_2(1-p_{22})},$$

$$U_2 = \gamma_{23}v_3 + \gamma_{22}v_2 = \frac{\lambda_3 q(1-p_{33})v_3 + \lambda_2 p_{22} v_2}{\lambda_3 q(1-p_{33}) + \lambda_2 p_{22}}.$$

Then the expected payoffs of types v_3 and v_2 if they solicit a rating are

$$R_3 = p_{33}\max\{U_3, \delta v_3\} + (1-p_{33})\max\{U_2, \delta v_3\},$$

$$R_2 = (1-p_{22})\max\{U_3, \delta v_2\} + p_{22}\max\{U_2, \delta v_2\}.$$

Note that these payoffs permit for the possibility that a rated issuer does not trade following a low rating.

The optimal rating system of the CRA under participation of types v_2 and v_3 is described in the following proposition.

Proposition 8 *If the target market coverage are issuer types v_2 and v_3, the CRA sets the information structure*

	v_3	v_2	v_1
s_3	1	$1-p_{22}$	0
s_2	0	p_{22}	0
s_1	0	0	1

where $p_{22} = \frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2} < 1$. It charges the rating fee $\phi = (1-\delta)\frac{\lambda_3 q v_3+\lambda_2 v_2}{\lambda_3 q+\lambda_2}$ and gains profits $\Pi_{2,3} = (1-\delta)\frac{(\lambda_2+\lambda_3)(q\lambda_3 v_3+\lambda_2 v_2)}{\lambda_3 q+\delta\lambda_2} < (1-\delta)E[v]$. Issuers types v_3 and v_2 solicit a rating, while issuer type v_1 does not solicit a rating. All rated issuers trade the asset. Informed investors obtain a positive rent equal to $\frac{\lambda_3\lambda_2(1-q)(1-\delta)(v_3-v_2)}{\lambda_3 q+\lambda_2}$.

The rating system involves rating "inflation" in the sense that while the highest type receives only the highest rating s_3, type v_2 can receive two ratings, s_2 and s_3. Since the CRA makes an optimistic "mistake" of assigning a higher rating s_3 to type v_2, it can be interpreted as rating inflation.

Although this information structure is optimal, there may be other optimal information structures, even if we restrict attention to three signals. In other words, inflation may not always be necessary for an optimal rating system. Below, we characterize the condition for inflation to be necessary and sufficient if the CRA chooses an information structure such that only types $i = 2, 3$ solicit a rating. A similar condition exists for the case where all three types solicit a rating.

Proposition 9 *Restrict attention to the set of equilibria in which only types v_2 and v_3 solicit a rating. Then for $\delta > \overline{\delta} = \frac{\lambda_2 v_2+q\lambda_3 v_3}{\lambda_2 v_3+q\lambda_3(2v_3-v_2)}$ equilibrium must entail rating inflation. As winner's curse problem becomes more severe, q decreases, the set of market conditions for which rating inflation is necessary increases, $\frac{d\overline{\delta}}{dq} > 0$.*

The result is similar to Proposition 5. High values of δ imply that the participation constraint of type v_3 is more costly to satisfy. Assuring that type v_3 is always assigned a high rating guarantees that it trades, and the surplus is not lost. As the winner's curse problem increases, the valuation of the issuer rated s_2 decreases, which makes rating inflation necessary.

Under market coverage of issuer types v_2 and v_3, the posterior beliefs of uninformed investors are such that ratings s_2 and s_1 perfectly reveal issuers types,[3] $\gamma_{11} = 1$ and $\gamma_{22} = 1$; rating s_3 leads to beliefs updating

$$\gamma_{33} = \frac{\lambda_3 q + \delta\lambda_2}{\lambda_3 q + \lambda_2} \text{ and } \gamma_{32} = \frac{(1-\delta)\lambda_2}{\lambda_3 q + \lambda_2}.$$

The CRA chooses the market coverage that provides the highest profit under a given set of market conditions. The next proposition explains the CRA's choice of the market coverage.

Proposition 10 *There exists an interval $[\overline{q}, 1]$ such that for all $q \in [\overline{q}, 1]$ the optimal rating system induces two issuer types v_3 and v_2 to solicit a rating; and for all $q \in [0, \overline{q}]$ it induces only type v_3 to solicit a rating.*

As the winner's curse problem becomes more severe, the CRA decreases the market coverage. The economic intuition of this result is as follows. Unless the information structure is perfectly informative, informed investors obtain the information rent. Thus the winner's curse reduces the surplus that can be captured by the CRA. Revealing more information reduces the adverse selection problem. However, more informative ratings also reduce the ability of the CRA to extract the surplus from the issuers. Under the market coverage with two types, the CRA's optimal information structure aims to increase the payoff of issuer type v_2 by pooling it with type v_3. Presence of informed investors leads to severe underpricing of an issue with the best rating s_3, which ultimately reduces the fee that the CRA can charge. As the winner's curse problem becomes substantial, the CRA is better off eliminating the underpricing by restricting market coverage to the best issuer type v_3.

When the CRA rates types v_2 and v_3, the information content of ratings depends on the market conditions. Given the information structure described in Proposition 8, the probability p_{22} that issuers type v_2 are assigned a rating s_2 can be interpreted as rating precision. Next proposition summarizes how the rating precision depends on the market conditions.

Proposition 11 *In the market with high share of uninformed investors, $q \in [\overline{q}, 1]$, the CRA reduces ratings precision*

(i) as the share of uninformed investors increases (q increases), $\frac{dp_{22}}{dq} < 0$;

[3] In equilibrium, investors never observe rating s_1 because issuers type v_1 do not solicit a rating.

(ii) as the aggregate value of liquidity increases (δ decreases), $\frac{dp_{22}}{d\delta} > 0$;
(iii) as high quality assets become more scares ($\frac{\lambda_2}{\lambda_3}$ increases), $\frac{dp_{22}}{d(\frac{\lambda_2}{\lambda_3})} < 0$.

In the extreme case, when all investors are uninformed ($q = 1$) and the value of liquidity is very high ($\delta = 0$), ratings are uninformative, $p_{22} = 0$.

The comparative statics results suggest that under the conditions of booming economy, that is, high share of uninformed investors and high value of liquidity, ratings are less informative. It may be an explanation for the poor performance of ratings of asset backed securities. In the pre-crisis period, these assets had higher returns relative to other securities. Also the period coincided with rapid growth in several developing countries that were eager to invest in ABS assets. The other result is that the information content of ratings depends on the distribution of investment opportunities. As high quality assets become more scarce, the CRA's major revenue is driven by rating intermediate type v_2. It provides incentives to reduce the precision of ratings.

6 Policy implications

In this section, we apply our theory to evaluate the effect of recent CRA reform proposals on ratings precision and the market outcome. We discuss the proposals on standardization of rating symbols, regulation of the rating fees, expert liability and reducing the reliance on ratings in regulation.

6.1 Standardization of rating symbols

Major rating agencies use rating symbols to communicate the credit quality of issuers to investors. Usually CRAs employ a dozen of rating categories and distinguish between investment grade and non-investment grade securities. The common practice is that the same rating symbols are applied to different asset classes rated by the same CRA. At the same time, the CRAs' rating methodology documents emphasize and the empirical evidence confirms that same rated securities from different asset classes may have different credit quality.

The difference of credit quality was especially stark for the asset backed securities that experienced massive downgrades during the 2007-2008 financial crisis. Combined with the fact that the majority of these securities were designed to have a high initial rating, the abrupt downgrades were followed by several policy proposals that aim to eliminate the potential investors' confusion about the credit risk of different asset classes. The

European Union regulators imposed the requirement that rating symbols of structured securities must have an additional "s" qualifier to identify the asset class. The Dodd-Frank Act in the US followed a different approach. It requested the SEC to conduct a study on standardization of rating symbols that would request that symbols for different asset classes correspond to the same credit quality. Also for the rating scale of a given rating class, the policy would require that different ratings have the same rating precision.

The effect of rating symbols standardization proposals can be evaluated within the scope of our model. Suppose that the CRA is required to provide the same accuracy for both asset classes, or across different ratings within the same rating class. Effectively it means that the CRA is restricted to a given rating precision $\overline{p}_{22}$ for different asset classes but has the flexibility to set the rating fees. The policy has the following effect.

Proposition 12 *Imposing rating standardization may decrease the market coverage and reduce market liquidity.*

Rating standardization limits the CRA's ability to design the rating system. Given precision level $\overline{p}_{22}$, the CRA optimizes the profits by adjusting the fee. If the required level of precision in a particular rating class exceed the optimal precision derived in Proposition 8, the CRA may choose to increase the fee so that only the highest quality issuers solicit a rating. This strategy can induce no trade for unrated v_2 issuers and leads to inefficiency.

The other rating standardization policy is to require CRA to provide the same precision for different ratings. In terms of our model, the CRA is required to set equal precision for the two types of issuers v_2 and v_3, $p_{22} = p_{33}$. Then the CRA's adjustment to the policy can take one of the following forms. It can provide ratings of high precision, $p_{ii} = 1$ but reduce the market coverage to the highest quality issuers, leading to illiquidity for issuers type v_2. Alternatively, it can sell ratings with precision $p_{ii} < 1$ to both types. However, it means that following a low rating, high quality issuers will refuse to trade. Thus it results in lower liquidity for high quality assets. Both outcomes reduce liquidity and lead to inefficiencies.

6.2 Regulation of rating fees

Rating agencies receive compensation for rating services from the issuers of securities or the parties participating in marketing the securities. Normally fee schedules are communicated to issuers prior to the issuance of a rating. The precise fee amounts are determined by various factors including the assets class of the rated security and the principal amount of the debt issuance that is rated. According to the code of conduct of the major NRSROs, the receipt of the compensation cannot influence the process of assigning a rating.

The rating fees have high variation across different asset classes. S&P US rating fees disclosure in 2008 indicates that the price of rating corporate debt was limited at 4.25 basis points while the structured finance fees ranged up to 12 basis points. In this section we analyze the effect of imposing a cap on the fee that a CRA may charge for rating a particular class of assets.

The effectiveness of the regulation that imposes a limit on the rating fee depends on the initial equilibrium outcome. If the market coverage involves rating two types v_2 and v_3, then limiting the fee does not change the optimal information structure. Indeed, given the fee, the CRA's objective is to maximize the market coverage. Then the CRA can choose to rate two or three types. In the former case, the trade occurs under the terms described in Proposition 8. In the later case of rating all types, the underpricing becomes more severe. It results in transfer of wealth from the CRA to the informed investors. However, in either case the original information structure remains optimal, and the policy has no effect on welfare.

The policy becomes effective when the original equilibrium involves limited market coverage. According to Proposition 7, this outcome occurs in the market with a high share of uninformed investors, $q > \overline{q}$. In this case, limiting the rating fee can improve efficiency.

Proposition 13 *Consider the market in which the CRA rates only one type v_3 and charges a fee ϕ. Imposing a fee cap $\overline{\phi} < \phi$ induces the CRA to rate two issuer types v_2 and v_3 and increases efficiency.*

In the market with a high share of uninformed investors, the CRA reduces the market coverage in order to eliminate the underpricing problem that would force it to reveal a lot of information. It charges a high fee that discourages participation of v_2 issuers. If a regulator can limit the fee to the level that is compatible with coverage of both types v_2 and v_3, the CRA's optimal reaction to the policy is to maximize the market coverage, which leads to the information structure described in Proposition 8. The regulation is efficient because it increases market liquidity by inducing issuers v_2 to trade.

6.3 Expert liability

Traditionally CRAs has been exempt from legal liability for inaccurate ratings under the First Amendment. The courts viewed ratings as an opinion about the credit quality.

There were several cases where CRAs were sued by investors when the credit quality of highly rated securities quickly deteriorated. However, the nature of the rating business makes it hard to demonstrate that the default could have been foreseen by the CRA at the time of assigning the rating.

Dodd-Frank Act has removed the First Amendment protection. Now investors can bring private rights of action against CRAs for a knowing or reckless failure to conduct a reasonable investigation of the facts or to obtain analysis from an independent source. Under Dodd- Frank Act, NRSROs are be subject to the same expert liability as auditors or security analysts. In particular, the new rules imply that the CRAs have to give their consent for their ratings report to be included in a new issue security prospectus. Facing higher legal risks, in many instances the CRAs refused to provide the consent.

What is the effect of introducing legal liability on the precision of ratings? Suppose that the CRA has to pay a fine when an issue rated s_3 realizes the value v_2. Then the following result holds.

Proposition 14 *Consider a market in which the CRA rates issuers types v_2 and v_3. Imposing a fine for overrating an issue increases ratings precision but may reduce market coverage to the highest type v_3.*

Imposing a fine makes rating inflation more costly, and thus increases the precision of ratings. However, more informative ratings reduce the ability of the CRA to extract the surplus as pooling type v_2 with the highest type v_3 has the liability cost. As a result, the CRA may choose to reduce the market coverage to the highest issuer type and provide precise rating, eliminating the legal liability risk. If this occurs, the market outcome is inefficient because the issuers type v_2 are not rated and may not trade.

6.4 Reliance on ratings in regulation

The US regulators has been using credit ratings from 1930s to control the risk taking behavior of regulated financial institutions and insurance companies. The regulatory use of ratings has expanded significantly starting the 1970s when the SEC adopted the rule which uses ratings as a basis for calculating capital requirements for broker-dealers. During the next thirty years the use of ratings in regulation has become common practice. National Association of Insurance Commissioners uses ratings to assess the risk of the insurance investment portfolio that ultimately determines the insurance company capital requirements. Department of Labor requires that the pension funds investment in asset-backed securities is restricted to securities rated A or higher. Investment grade mutual funds must sell any security rated B or below, and cannot hold more than 5% of non-investment grade securities. The Basel II proposes to use credit ratings to determine the securitization exposures for banks.

Rating based regulatory policies effectively imposed a regulatory premium on higher rated bonds and potentially decrease the liquidity on the market of lower rated bonds. The CRAs have been criticized for providing the "regulatory licence" instead of unbiased credit analysis. Following the crisis, the regulators in the US and in the EU discussed several policy options to reduce the regulatory reliance on ratings. So far, the task has been challenging as there are few alternatives that can substitute ratings in prudential regulation.

Consistent with the observations of the investment community, our model predicts that regulatory use of ratings reduces their precision. Indeed, suppose that an issuer gains a regulatory premium $\rho > 0$ if an issue obtains the highest rating s_3. Then the CRA will adjust its rating system to extract the regulatory rent.

Proposition 15 *Consider a market where issuers types v_2 and v_3 are rated. Introducing a regulatory premium ρ for securities rated s_3 reduces the precision of ratings.*

The economic intuition of this result is the following. In the basic model, the reason for rating inflation is that the CRA increases the willingness to pay of issuers v_2 by assigning them a high rating with probability $1 - p_{22}$. If the high rating value is increased by the regulatory premium, it makes rating inflation more desirable for the CRA. It is also feasible because high quality issuers value of trade is increased by regulation, and they are willing to accept less precise ratings. The regulatory premium increases the profits of the CRA. Also it permits informed investors to gain higher rent as lower rating precision aggravates the winner's curse problem.

7 Conclusion

In this paper we analyze the equilibrium precision of ratings. Our results suggest that the information content of ratings depends on the market conditions and the presence of differentially informed investors. In particular, we show that as the share of uninformed investors and the aggregate value of liquidity increase, ratings become less informative. Also we show that the ratings become less informative when the share of high quality assets in the economy decreases. The results offer an explanation for heterogenous performance of ratings in different asset classes and through the cycle. We apply the model to analyze the merits of the recent reform proposals and show that some policies, in particular, rating standardization and expert liability, reduce market efficiency.

Understanding the incentives of CRAs to produce information is important for guiding policies to improve the efficiency of the financial market. Several aspect of the problem are left for future research. The focus of the paper was to evaluate the performance of ratings under the current issuer pays model. Many commentators in academic and investment communities suggest that conversion to investor pays model may increase ratings precision. It is unclear, however, whether switching the side will lead to better ratings or shift the CRAs incentives to provide ratings that are biased in favor of investors' needs. Also our focus was on a monopoly CRA. High industry concentration has recently led to Duopoly Relief Act of 2006 that aims to encourage competition among CRAs. The effect of competition on the information content of ratings is another area that needs further analysis.

Appendix: Proofs

Proof of Proposition 1. Suppose that the proposition does not hold. Without loss of generality, there must exist types k, j such that $R_k - \delta v_k > R_j - \delta v_j = \phi$, and yet the CRA cannot increase its profit by increasing R_j and decreasing R_k. Then we claim that we must have $k > j$. To see this, first, note that R_k is a continuos function of p_{ij} for all k, i and j. Furthermore, perfectly informative signals, lead to $R_j = v_j$ so that $R_k - \delta v_k > R_j - \delta v_j$ implies $k > j$. However, if $k > j$, then we can make the signals more uninformative until $R_k - \delta v_k = R_j - \delta v_j$ since (i) R_k is a continuos function of p_{ij} for all k, i and j; and (ii) fully noisy signals result in $R_k - \delta v_k < R_j - \delta v_j$ as long as $k > j$. Therefore, this completes our proof. ■

Proof of Proposition 2. If all three types are rated, then

$$\sum_j \lambda_j R_j = \sum_j \lambda_j v_j = E[v].$$

From Proposition 1,

$$\begin{aligned}\sum_j \lambda_j R_j &= \lambda_3(R_1 + \delta v_3) + \lambda_2(R_1 + \delta v_2) + R_1 \\ &= R_1 + \delta(\lambda_3 v_3 + \lambda_2 v_2) = E[v].\end{aligned}$$

Thus,

$$\begin{aligned}\phi &= R_1 = E[v] - \delta(\lambda_3 v_3 + \lambda_2 v_2) = (1-\delta)E[v], \\ \pi^3 &= (1-\delta)E[v],\end{aligned}$$

where π^3 denotes the profit of the CRA when all three types are rated.

If two types v_3 and v_2 are rated, Proposition 1 obtains that

$$R_2 - \delta v_2 = R_3 - \delta v_3 = \phi,$$

$$\lambda_3 R_3 + \lambda_2 R_2 = E[v].$$

Thus,

$$\begin{aligned}(\lambda_3 + \lambda_2)R_2 &= E[v] - \lambda_3 \delta(v_3 - v_2), \\ \pi^2 &= (\lambda_3 + \lambda_2)\phi = (\lambda_3 + \lambda_2)R_2 - (\lambda_3 + \lambda_2)\delta v_2 \\ &= (1-\delta)E[v],\end{aligned}$$

where π^2 denotes the CRA's profit when types v_3 and v_2 are rated. Hence, $\pi^2 = \pi^3$ and the CRA is indifferent between rating two or three types.

If one type v_3 is rated, then Proposition 1 obtains that

$$R_3 - \delta v_3 = \phi,$$

$$R_3 = v_3,$$

$$\pi^1 = (1-\delta)\lambda_3 v_3 < \pi^2 = \pi^3.$$

There exists a rating system that implements R_j. Indeed, consider the rating system with $p_{33} = p_{11} = 1$, $p_{22} = \frac{\delta(\lambda_3+\lambda_2)}{\lambda_3+\delta\lambda_2}$, and $p_{ij} = 0$ for the other i, j. It is immediate to verify that this system implements R_j and permits the CRA to extract the surplus. ■

Proof of Proposition 3. From Proposition 2, the issuers willingness to pay for ratings is a weighted average of the expected value of the asset and the issuer's type,

$$R_i^2 = (1-\delta)\frac{E[v]}{\lambda_2+\lambda_3} + \delta v_i, \ i = 2, 3.$$

Thus as the value of outside option increases, the rating has to be more informative about the issuer's type. ■

Proof of Proposition 6. Consider a general information structure that leads to posterior distribution $\beta_{ij} = \Pr(v_j, s_i)$. Denote $\gamma_{ij} = \Pr(v_j | s_i)$ the belief of an uninformed investor who is offered an issue that the issuer rated s_i is of type v_j.

We consider two cases where the CRA rates (i) all types, or (ii) two highest types v_3 and v_2.

Case (i). Suppose the CRA rates all issuers. Denote b_i^* the equilibrium price of a security rated s_i. Two cases are possible, $b_i^* \in (v_1, v_2)$ and $b_i^* \in (v_2, v_3)$.

If $b_i^* \in (v_1, v_2)$, the posterior beliefs of uninformed investors are

$$\begin{aligned}
\gamma_{i3} &= \frac{q\beta_{i3}}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}}, \\
\gamma_{i2} &= \frac{q\beta_{i2}}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}}, \\
\gamma_{i1} &= \frac{\beta_{i1}}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}}.
\end{aligned}$$

Then

$$b_i^* = \Sigma_j \gamma_{ij} v_j = \frac{q\beta_{i3}v_3 + q\beta_{i2}v_2}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}},$$

and $b_i^* \in (v_1, v_2)$ holds when

$$\frac{q\beta_{i3}v_3 + q\beta_{i2}v_2}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} < v_2,$$

which simplifies to

$$\frac{q\beta_{i3}}{\beta_{i1}} < \frac{v_2}{v_3 - v_2}.$$

The surplus is equal to

$$\frac{q\beta_{i3}v_3 + q\beta_{i2}v_2}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}}.$$

If $b_i^* \in (v_2, v_3)$, then the posterior beliefs of uninformed investors are

$$\begin{aligned}
\gamma_{i3} &= \frac{q\beta_{i3}}{q\beta_{i3} + \beta_{i2} + \beta_{i1}}, \\
\gamma_{i2} &= \frac{\beta_{i2}}{q\beta_{i3} + \beta_{i2} + \beta_{i1}}, \\
\gamma_{i1} &= \frac{\beta_{i1}}{q\beta_{i3} + \beta_{i2} + \beta_{i1}}.
\end{aligned}$$

Then

$$b_i^* = \Sigma_j \gamma_{ij} v_j = \frac{q\beta_{i3}v_3 + \beta_{i2}v_2}{q\beta_{i3} + \beta_{i2} + \beta_{i1}},$$

and $b_i^* \in (v_2, v_3)$ holds when

$$\frac{q\beta_{i3}v_3 + \beta_{i2}v_2}{q\beta_{i3} + \beta_{i2} + \beta_{i1}} > v_2$$

which simplifies to

$$\frac{q\beta_{i3}}{\beta_{i1}} > \frac{v_2}{v_3 - v_2}.$$

The surplus is equal to

$$\frac{q\beta_{i3}v_3 + \beta_{i2}v_2}{q\beta_{i3} + \beta_{i2} + \beta_{i1}}.$$

Case (ii). When only two higher types are rated, the price must satisfy $b_i^* \in (v_2, v_3)$. The posterior beliefs are

$$\begin{aligned}
\gamma_{i3} &= \frac{q\beta_{i3}}{q\beta_{i3} + \beta_{i2}}, \\
\gamma_{i2} &= \frac{\beta_{i2}}{q\beta_{i3} + \beta_{i2}}.
\end{aligned}$$

Then

$$b_i^* = \Sigma_j \gamma_{ij} v_j = \frac{q\beta_{i3}v_3 + \beta_{i2}v_2}{q\beta_{i3} + \beta_{i2}}.$$

The surplus is equal to

$$(\lambda_3+\lambda_2)b_i^* = \frac{(\lambda_3+\lambda_2)(q\beta_{i3}v_3+\beta_{i2}v_2)}{q\beta_{i3}+\beta_{i2}}.$$

The price satisfies $b_i^* \in (v_2, v_3)$ for all parameter values.

Next, we consider any signal s_i and show that the expected revenue with two ratings is higher than that with three ratings for each signal.

If $b_i^* \in (v_2, v_3)$, then we need to show that

$$\frac{q\beta_{i3}v_3+\beta_{i2}v_2}{q\beta_{i3}+\beta_{i2}+\beta_{i1}} < \frac{(\beta_{i3}+\beta_{i2})(q\beta_{i3}v_3+\beta_{i2}v_2)}{q\beta_{i3}+\beta_{i2}},$$
$$\text{or } \frac{(\beta_{i3}+\beta_{i2})(q\beta_{i3}+\beta_{i2}+\beta_{i1})}{q\beta_{i3}+\beta_{i2}} > 1.$$

Denote $A = q\beta_{i3}+\beta_{i2}+\beta_{i1}$. Then the condition writes

$$\begin{aligned}\frac{(1-\beta_{i1})A}{A-\beta_{i1}} &> 1,\\ \text{or } A &< 1,\end{aligned}$$

which holds for all parameter values as long as $q < 1$.

If $b_i^* \in (v_1, v_2)$, then we need to show that

$$\frac{q\beta_{i3}v_3+\beta_{i2}v_2}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}} < \frac{(\beta_{i3}+\beta_{i2})(q\beta_{i3}v_3+\beta_{i2}v_2)}{q\beta_{i3}+\beta_{i2}}$$

holds for all parameter values such that $\frac{q\beta_{i3}}{\beta_{i1}} < \frac{v_2}{v_3-v_2}$ and $v_3 > v_2$, or equivalently, $v_3 \in (v_2, \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2)$. When we have $v_3 = \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2$, then $b_i^* = v_2$. In other words, for $v_3 = \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2$ we have the left hand side (LHS) is equal to v_2. For $v_3 = \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2$ the right hand side (RHS) is

$$\frac{(\beta_{i3}+\beta_{i2})(q\beta_{i3}\frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2+\beta_{i2}v_2)}{q\beta_{i3}+\beta_{i2}}$$

Simplifying leads to

$$\frac{(\beta_{i3}+\beta_{i2})(q\beta_{i3}+\beta_{i2}+\beta_{i1})v_2}{q\beta_{i3}+\beta_{i2}}.$$

Let $B = q\beta_{i3}+\beta_{i2}$. Then the condition becomes

$$\frac{(1-\beta_{i1})(B+\beta_{i1})v_2}{B}.$$

Given that $\frac{(1-\beta_{i1})(B+\beta_{i1})v}{B} > 1$ simplifies to $q\beta_{i1} + \beta_{i2} + \beta_{i3} < 1$, we have RHS greater than v_2.

The other extreme condition under $v_3 \in (v_2, \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}} v_2)$ is $v_3 = v_2$. Then LHS becomes $\frac{(q\beta_{i3}+q\beta_{i2})v_2}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}}$ and RHS becomes $\frac{(\beta_{i3}+\beta_{i2})(q\beta_{i3}+\beta_{i2})v_2}{q\beta_{i3}+\beta_{i2}}$. For our result to hold for this extreme point, we must have

$$\frac{q\beta_{i3} + q\beta_{i2}}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} < \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3} + \beta_{i2})}{q\beta_{i3} + \beta_{i2}}$$

This term simplifies to

$$\frac{q}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} < 1$$

Further iteration leads to $q < q(\beta_{i3} + \beta_{i2}) + \beta_{i1}$. Subtracting $(1-q)\beta_{i1}$ from both sides we have $q - (1-q)\beta_{i1} < q(\beta_{i3} + \beta_{i2} + \beta_{i1})$. Since $\beta_{i3} + \beta_{i2} + \beta_{i1} = 1$, we must have LHS less than RHS for $v_3 = v_2$.

Next we show that the difference between LHS and RHS is monotonic, and thus LHS is less than RHS for all interior points, $v_3 \in (v_2, \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}} v_2)$.

Define

$$F(v_3, a) = \frac{q\beta_{i3}v_3 + q\beta_{i2}v_2}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} - \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3}v_3 + \beta_{i2}v_2)}{q\beta_{i3} + \beta_{i2}},$$

where a denotes all the parameters, $a = (\lambda_H, \lambda_M, q)$. Then

$$\begin{aligned} F'_{v_3}(v_3, a) &= \frac{q\beta_{i3}}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} - \frac{(\beta_{i3} + \beta_{i2})q\beta_{i3}}{q\beta_{i3} + \beta_{i2}} \\ &= \frac{q\beta_{i3}}{(q(\beta_{i3} + \beta_{i2}) + \beta_{i1})(q\beta_{i3} + \beta_{i2})}(q\beta_{i3} + \beta_{i2} - (\beta_{i3} + \beta_{i2})(q(\beta_{i3} + \beta_{i2}) + \beta_{i1})) \end{aligned}$$

The first term is positive, thus the sign of $F'_{v_3}(v_3, a)$ is defined by the sign of

$$\begin{aligned} & q\beta_{i3} + \beta_{i2} - (\beta_{i3} + \beta_{i2})(q(\beta_{i3} + \beta_{i2}) + \beta_{i1}) \\ = \; & (1-q)(\beta_{i3} - 2(\frac{1}{2} - \beta_{i2})\beta_{i3} + \beta_{i2}^2). \end{aligned}$$

Solve the inequality with respect to β_{i3},

$$\beta_{i3}^2 - 2(\frac{1}{2} - \beta_{i2})\beta_{i3} + \beta_{i2}^2 > 0. \tag{3}$$

The roots are

$$\frac{1}{2} - \beta_{i2} \pm \sqrt{(\frac{1}{2} - \beta_{i2})^2 - \beta_{i2}^2} = \frac{1}{2} - \beta_{i2} \pm \sqrt{\frac{1}{4} - \beta_{i2}}.$$

If $\beta_{i2} > \frac{1}{4}$, then (3) holds for any $a \in A_1 = \{\beta_{i3} \in (0,1), \beta_{i2} \in (\frac{1}{4}, 1), q \in (0,1)\}$.

If $\beta_{i2} \leq \frac{1}{4}$, then note that the low root is positive, $\frac{1}{2} - \beta_{i2} - \sqrt{\frac{1}{4} - \beta_{i2}} > 0$ and the high root is less than 1, $\frac{1}{2} - \beta_{i2} + \sqrt{\frac{1}{4} - \beta_{i2}} < 1$. Therefore, if $a \in A_2$, where

$$A_2 = \{\beta_{i2} \in (0, \frac{1}{4}), \beta_{i3} \in (0, \frac{1}{2} - \beta_{i2} - \sqrt{\frac{1}{4} - \beta_{i2}}) \cup (\frac{1}{2} - \beta_{i2} + \sqrt{\frac{1}{4} - \beta_{i2}}, 1), q \in (0,1)\},$$

then $F' > 0$. If $a \in A_3$, where

$$A_3 = \{\beta_{i2} \in (0, \frac{1}{4}), \beta_{i3} \in (\frac{1}{2} - \beta_{i2} - \sqrt{\frac{1}{4} - \beta_{i2}}, \frac{1}{2} - \beta_{i2} + \sqrt{\frac{1}{4} - \beta_{i2}}), q \in (0,1)\},$$

then $F' < 0$.

Hence, for $a \in A_1 \cup A_2$, $F' > 0$, and $F(\frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}} v_2, a) < 0$ implies the result. For $a \in A_3$, $F' < 0$ and $F(v_2, a) < 0$ implies the result.

Considering the fact that the expected revenue with two ratings is higher than that with three ratings for any signal, the expected revenue for any issuer type is greater with two ratings. Furthermore, an analogue for Proposition 1 hold for $q < 1$ case as well. Therefore, the CRA's expected profit is higher when the CRA rates two highest types rather than all types.

Consequently, we are left to show that rating all types is never an equilibrium. Suppose that such an equilibrium exists. Then consider the following deviation by the CRA. CRA deviates to an information structure that is identical except an addition of a new signal that can be received only by v_1. By above arguments, this information structure leads to higher expected revenues as only two highest types can have positive revenues. Furthermore, the CRA can extract all the revenues by optimally choosing a new information structure and a fee $\phi = R_2 - \delta v_2 = R_3 - \delta v_3$. Therefore, for any candidate equilibrium with all types rated, there exists a profitable deviation. ■

Proof of Proposition 7. The proof is in text. ■

Proof of Proposition 8. Consider the information structure

	v_3	v_2	v_1
s_3	p_{33}	$1-p_{22}$	0
s_2	$1-p_{33}$	p_{22}	0
s_1	0	0	1

Under this information structure, rating s_1 perfectly reveals the types v_1, $\gamma_{13} = \gamma_{12} = 0$,

$\gamma_{11} = 1$. Ratings s_2 and s_3 lead to beliefs updating $\gamma_{ij} = \Pr(v_j | \, s_i)$, with

$$\begin{aligned}
\gamma_{33} &= \frac{\lambda_3 q p_{33}}{\lambda_3 q p_{33} + \lambda_2(1 - p_{22})}, \\
\gamma_{32} &= \frac{\lambda_2(1 - p_{22})}{\lambda_3 q p_{33} + \lambda_2(1 - p_{22})}, \\
\gamma_{31} &= 0.
\end{aligned}$$

$$\begin{aligned}
\gamma_{23} &= \frac{\lambda_3 q(1 - p_{33})}{\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22}}, \\
\gamma_{22} &= \frac{\lambda_2 p_{22}}{\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22}}, \\
\gamma_{21} &= 0.
\end{aligned}$$

Under this information structure, the issuers expected payoff if they solicit a rating are equal to

$$\begin{aligned}
R_3 &= p_{33} \max\{U_3, \delta v_3\} + (1 - p_{33}) \max\{U_2, \delta v_3\}, \\
R_2 &= (1 - p_{22}) \max\{U_3, \delta v_2\} + p_{22} \max\{U_2, \delta v_2\}, \\
R_1 &= v_1 = 0.
\end{aligned}$$

The payoff of type v_1 implies that it does not solicit a rating.

The investor's assessment of the asset values under this information structure is

$$U_3 = \gamma_{33} v_3 + \gamma_{32} v_2 = \frac{\lambda_3 q p_{33} v_3 + \lambda_2(1 - p_{22}) v_2}{\lambda_3 q p_{33} + \lambda_2(1 - p_{22})},$$

$$U_2 = \gamma_{23} v_3 + \gamma_{22} v_2 = \frac{\lambda_3 q(1 - p_{33}) v_3 + \lambda_2 p_{22} v_2}{\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22}}.$$

Consider the case when all rated issuers trade, $U_i \geq \delta v_j$ for all $i, j = 2, 3$. (Note that if $U_i < \delta v_j$ for all i, j, then types v_j do not solicit a rating.) Then the optimal information structure solves

$$\max \ (\lambda_3 + \lambda_2)\phi$$

$$\begin{aligned}
\rho_3 :&\quad p_{33} U_3 + (1 - p_{33}) U_2 - \delta v_3 - \phi \geq 0, \\
\rho_2 :&\quad (1 - p_{22}) U_3 + p_{22} U_2 - \delta v_2 - \phi \geq 0, \\
\mu_3 :&\quad p_{33}(U_3 - \delta v_3) \geq 0, \\
\mu_2 :&\quad (1 - p_{33})(U_2 - \delta v_3) \geq 0, \\
\tau_3 :&\quad 1 - p_{33} \geq 0, \\
\tau_2 :&\quad 1 - p_{22} \geq 0, \\
\kappa_3 :&\quad p_{33} \geq 0, \\
\kappa_2 :&\quad p_{22} \geq 0.
\end{aligned}$$

Note that we did not include the constraints $U_i - \delta v_2 \geq 0$ because these are implied by the other two constraints.

The first order conditions of the problem are:

with respect to p_{33}

$$\rho_3(U_3 - U_2 + p_{33}\frac{dU_3}{dp_{33}} + (1-p_{33})\frac{dU_2}{dp_{33}}) + \rho_2((1-p_{22})\frac{dU_3}{dp_{33}} + p_{22}\frac{dU_2}{dp_{33}})$$
$$+\mu_3(U_3 - \delta v_3 + p_{33}\frac{dU_3}{dp_{33}}) + \mu_2(-(U_2 - \delta v_3) + (1-p_{33})\frac{dU_2}{dp_{33}}) - \tau_3 + \kappa_3 = 0,$$

with respect to p_{22}

$$\rho_3(p_{33}\frac{dU_3}{dp_{22}} + (1-p_{33})\frac{dU_2}{dp_{22}}) + \rho_2(U_2 - U_3 + (1-p_{22})\frac{dU_3}{dp_{22}} + p_{22}\frac{dU_2}{dp_{22}})$$
$$+\mu_3 p_{33}\frac{dU_3}{dp_{22}} + \mu_2(1-p_{33})\frac{dU_2}{dp_{22}} - \tau_2 + \kappa_2 = 0,$$

with respect to ϕ

$$(\lambda_3 + \lambda_2) - \rho_3 - \rho_2 = 0.$$

In terms of the information structure, the values $\frac{dU_i}{dp_{kk}}$, $i,k = 2,3$ write

$$\frac{dU_3}{dp_{33}} = \frac{\lambda_3\lambda_2 q(1-p_{22})(v_3 - v_2)}{(\lambda_3 q p_{33} + \lambda_2(1-p_{22}))^2} > 0,$$

$$\frac{dU_3}{dp_{22}} = \frac{\lambda_3\lambda_2 q p_{33}(v_3 - v_2)}{(\lambda_3 q p_{33} + \lambda_2(1-p_{22}))^2} > 0,$$

$$\frac{dU_2}{dp_{33}} = -\frac{\lambda_3\lambda_2 q p_{22}(v_3 - v_2)}{(\lambda_3 q(1-p_{33}) + \lambda_2 p_{22})^2} < 0,$$

$$\frac{dU_2}{dp_{22}} = -\frac{\lambda_3\lambda_2 q(1-p_{33})(v_3 - v_2)}{(\lambda_3 q(1-p_{33}) + \lambda_2 p_{22})^2} < 0.$$

Proposition 1 implies that rated issuers must have the same willingness to pay, and thus $\rho_i > 0$, $i = 2,3$. Then

$$(p_{33} - (1-p_{22}))(U_3 - U_2) = \delta(v_3 - v_2). \tag{4}$$

Case (a). Suppose that $\mu_i > 0$ for $i = 2,3$. Then three cases are possible, (a1) $p_{33} = 0$ and $U_2 = \delta v_3$; (a2) $U_3 = \delta v_3$ and $p_{33} = 1$; (a3) $U_3 = U_2 = \delta v_3$. Case (a1) is not feasible due to (4) and $U_3 \geq \delta v_3$. Case (a2) is not feasible due to (4) and $p_{22} \leq 1$. Case (a3) is not feasible due to (4). Thus $\mu_i > 0$ for $i = 1,2$ is not a solution.

Case (b). Suppose that $\mu_3 = 0$ and $\mu_2 > 0$. Thus the optimal information structure is determined by the conditions

$$(1 - p_{33})(U_2 - \delta v_3) = 0,$$
$$(p_{33} - (1 - p_{22}))(U_3 - U_2) = \delta(v_3 - v_2).$$

We obtain

$$U_3 - U_2 = \frac{\lambda_3 \lambda_2 q(p_{33}p_{22} - (1 - p_{33})(1 - p_{22}))}{(\lambda_3 q p_{33} + \lambda_2(1 - p_{22}))(\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22})}(v_3 - v_2),$$

and thus

$$\frac{\lambda_3 \lambda_2 q(p_{33} - (1 - p_{22}))^2}{(\lambda_3 q p_{33} + \lambda_2(1 - p_{22}))(\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22})} = \delta.$$

Two alternatives are possible for $(1 - p_{33})(U_2 - \delta v_3) = 0$. Consider first the case $p_{33} = 1$. Then

$$\frac{\lambda_3 q p_{22}}{\lambda_3 q + \lambda_2(1 - p_{22})} = \delta,$$
$$\lambda_3 q p_{22} - \delta(\lambda_3 q + \lambda_2(1 - p_{22})) = 0$$
$$p_{22}(\lambda_3 q + \delta\lambda_2) - \delta(\lambda_3 q + \lambda_2) = 0$$
$$p_{22} = \frac{\delta(\lambda_3 q + \lambda_2)}{\lambda_3 q + \delta\lambda_2}.$$

Then the values $\frac{dU_i}{dp_{kk}}$ write

$$\frac{dU_3}{dp_{33}} = \frac{\lambda_3 \lambda_2 q(1 - p_{22})(v_3 - v_2)}{(\lambda_3 q + \lambda_2(1 - p_{22}))^2},$$
$$\frac{dU_3}{dp_{22}} = \frac{\lambda_3 \lambda_2 q(v_3 - v_2)}{(\lambda_3 q + \lambda_2(1 - p_{22}))^2},$$
$$\frac{dU_2}{dp_{33}} = -\frac{\lambda_3 q(v_3 - v_2)}{\lambda_2 p_{22}} < 0,$$
$$\frac{dU_2}{dp_{22}} = 0.$$

The values U_i write

$$U_3 = \frac{\lambda_3 q v_3 + \lambda_2(1 - p_{22})v_2}{\lambda_3 q + \lambda_2(1 - p_{22})},$$
$$U_2 = v_2.$$

We need to verify that this solution satisfies ex-ante and ex-post participation constraints. Constraints with respect to $\rho_i > 0$ for $i = 2, 3$ imply

$$\begin{array}{l} U_3 - \delta v_3 - \phi = 0, \\ (1 - p_{22})U_3 + p_{22}U_2 - \delta v_2 - \phi = 0, \end{array}$$

Thus

$$\begin{aligned} \phi &= U_3 - \delta v_3 \\ &= \frac{\lambda_3 q v_3 + \lambda_2(1 - p_{22})v_2}{\lambda_3 q + \lambda_2(1 - p_{22})} - \delta v_3, \end{aligned}$$

and then

$$p_{22}(U_3 - U_2) - \delta(v_3 - v_2) = 0,$$

which holds. The ex-post constraints are

$$\begin{aligned} U_3 - \delta v_3 &> 0, \\ \lambda_3 q v_3 + \lambda_2 v_2 \frac{\lambda_3 q(1-\delta)}{\lambda_3 q + \delta\lambda_2} - \delta v_3(\lambda_3 q + \lambda_2(1 - \frac{\delta(\lambda_3 q + \lambda_2)}{\lambda_3 q + \delta\lambda_2})) &> 0, \\ \lambda_3 q v_3 + \lambda_2 v_2 &> 0. \end{aligned}$$

The CRA charges the fee

$$\phi = U_3 - \delta v_3 = (1-\delta)\frac{\lambda_3 q v_3 + \lambda_2 v_2}{\lambda_3 q + \lambda_2}.$$

it gains profit

$$(\lambda_2 + \lambda_3)\phi = (1-\delta)(\lambda_2 + \lambda_3)\frac{\lambda_3 q v_3 + \lambda_2 v_2}{\lambda_3 q + \lambda_2}.$$

The surplus of informed investors is equal to the difference between the ex-ante market surplus and the CRA's profits,

$$\lambda_2 v_2 + \lambda_3 v_3 - (\lambda_2 + \lambda_3)\phi = \frac{\lambda_3\lambda_2(1-q)(1-\delta)(v_3 - v_2)}{\lambda_3 q + \lambda_2}.$$

Type v_3 obtains the rating s_2 with zero probability, thus the ex-post constraint is satisfied,

$$(1 - p_{33})(U_2 - \delta v_3) = 0.$$

The ex-post constraint for the type v_2 following rating s_3 is implied by the constraint for type v_3. The ex-post constraint for type v_2 following rating s_2 is $(1-\delta)v_2 > 0$.

It is immediate to verify that any rating system that permits no trade following the rating reduces the CRA's profits. ■

Proof of Proposition 9. Let $p_{33} = p < 1$ so that we simplify our notation for this derivation. $R_3 = pU_3 + (1-p)U_2$ and $U_3 = v_3$ and $U_2 = \frac{\lambda_3(1-p)qv_3+\lambda_2 v_2}{\lambda_3(1-p)q+\lambda_2}$. Therefore, we have $R_3 = pv_3 + \frac{\lambda_3(1-p)^2qv_3+\lambda_2(1-p)v_2}{\lambda_3(1-p)q+\lambda_2}$ and $R_2 = U_2 = \frac{\lambda_3(1-p)qv_3+\lambda_2 v_2}{\lambda_3(1-p)q+\lambda_2}$. Then the first order condition is $pv_3 + \frac{\lambda_3(1-p)^2qv_3+\lambda_2(1-p)v_2}{\lambda_3(1-p)q+\lambda_2} - \delta v_3 = \frac{\lambda_3(1-p)qv_3+\lambda_2 v_2}{\lambda_3(1-p)q+\lambda_2} - \delta v_2$. Solution is $p = \frac{\delta(\lambda_2+q\lambda_3)}{\lambda_2+q\delta\lambda_3}$.

One can show that the highest value of U_2 occurs for the deflation case if we restrict attention to optimal R_i. Therefore, for participation condition we must have $U_2 = \frac{\lambda_3(1-p)qv_3+\lambda_2 v_2}{\lambda_3(1-p)q+\lambda_2} \geq \delta v_3$ so for cut off $\delta = \frac{(1-p)\lambda_3 qv_3+\lambda_2 v_2}{(1-p)\lambda_3 qv_3+\lambda_2 v_3}$ substituting $p = \frac{\delta(\lambda_2+q\lambda_3)}{\lambda_2+q\delta\lambda_3}$ we get $\delta = \frac{(1-\frac{\delta(\lambda_2+q\lambda_3)}{\lambda_2+q\delta\lambda_3})\lambda_3 qv_3+\lambda_2 v_2}{(1-\frac{\delta(\lambda_2+q\lambda_3)}{\lambda_2+q\delta\lambda_3})\lambda_3 qv_3+\lambda_2 v_3}$. Then the solution is $\overline{\delta} = \frac{\lambda_2 v_2+q\lambda_3 v_3}{\lambda_2 v_3 - q\lambda_3 v_2 + 2q\lambda_3 v_3} = \frac{\lambda_2 v_2+q\lambda_3 v_3}{\lambda_2 v_3+q\lambda_3(2v_3-v_2)}$.

Taking the derivative of $\overline{\delta}$ with respect to q, we obtain

$$\frac{\lambda_3 v_3(\lambda_2 v_3 + q\lambda_3(2v_3 - v_2)) - \lambda_3(2v_3 - v_2)(\lambda_2 v_2 + q\lambda_3 v_3)}{(\lambda_2 v_3 + q\lambda_3(2v_3 - v_2))^2}$$

The numerator simplifies to $\lambda_2\lambda_3 (v_2 - v_3)^2 > 0$. Therefore, as the winner's curse increases, $\overline{\delta}$ decreases leading to a larger set of δ that requires inflation. ■

Proof of Proposition 10. In the optimal information structure with types v_2 and v_3 rated we have $p_{22} = \frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2}$, $\gamma_{33} = \frac{\lambda_3 q+\delta\lambda_2}{\lambda_3 q+\lambda_2}$ and $\gamma_{32} = \frac{(1-\delta)\lambda_2}{\lambda_3 q+\lambda_2}$.Then the expected revenue is

$$\lambda_3(1 + \frac{\lambda_3 q(1-\delta)}{\lambda_3 q+\delta\lambda_2})\left(\frac{\lambda_3 q+\delta\lambda_2}{\lambda_3 q+\lambda_2}v_3 + \frac{(1-\delta)\lambda_2}{\lambda_3 q+\lambda_2}v_2\right) + \lambda_2\frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2}v_2$$

The profits of CRA π^2 will be

$$\lambda_3(1 + \frac{\lambda_3 q(1-\delta)}{\lambda_3 q+\delta\lambda_2})\left(\frac{\lambda_3 q+\delta\lambda_2}{\lambda_3 q+\lambda_2}v_3 + \frac{(1-\delta)\lambda_2}{\lambda_3 q+\lambda_2}v_2\right) + \lambda_2\frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2}v_2 - \delta(\lambda_3 v_3 + \lambda_2 v_2)$$

Recall that the profits for rating only type 3 is $\pi^1 = \lambda_3 \min\{(1-\delta)v_3, v_3 - \frac{\lambda_2 v_2}{\lambda_1+\lambda_2}\}$. We need to solve for q such that $\pi^2 = \pi^1$. Therefore, we have to solve for q: $\lambda_3(1 + \frac{\lambda_3 q(1-\delta)}{\lambda_3 q+\delta\lambda_2})\left(\frac{\lambda_3 q+\delta\lambda_2}{\lambda_3 q+\lambda_2}v_3 + \frac{(1-\delta)\lambda_2}{\lambda_3 q+\lambda_2}v_2\right) + \lambda_2\frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2}v_2 - \delta(\lambda_3 v_3 + \lambda_2 v_2) = \lambda_3 \min\{(1-\delta)v_3, v_3 - \frac{\lambda_2 v_2}{\lambda_1+\lambda_2}\}$.

As the winner's curse problem increases, we obtain

$$\begin{aligned}
\lim_{q\to 0} \pi^2 &= \lambda_3 (\delta v_3 + (1-\delta)v_2) + \lambda_2 v_2 - \delta(\lambda_3 v_3 + \lambda_2 v_2) \\
&= \lambda_3\delta v_3 + \lambda_3(1-\delta)v_2 + \lambda_2 v_2 - \delta\lambda_3 v_3 - \delta\lambda_2 v_2.
\end{aligned}$$

Simplifying leads to $\lambda_3(1-\delta)v_2 + \lambda_2 v_2(1-\delta) = v_2(1-\delta)(\lambda_3+\lambda_2)$. ■

Proof of Proposition 11. We obtain the following comparative statics results.

As the share of uninformed investors increases, the ratings become less informative.

$$\frac{dp_{22}}{dq} = -\frac{\lambda_3\lambda_2\delta(1-\delta)}{(\lambda_3 q+\delta\lambda_2)^2} < 0.$$

As the aggregate value of liquidity increases, that is δ decreases, the ratings become less informative.

$$\frac{dp_{22}}{d\delta} = \frac{\delta\lambda_2(\lambda_3 q+\lambda_2)}{(\lambda_3 q+\delta\lambda_2)^2} > 0.$$

As high quality assets become more scares, $\frac{\lambda_2}{\lambda_3}$ increases, ratings become less informative. Define $s = \frac{\lambda_2}{\lambda_3}$. Then $p_{22} = \frac{\delta(q+s)}{q+\delta s}$ and

$$\frac{dp_{22}}{ds} = -\frac{\delta(1-\delta)q}{(q+\delta s)^2} < 0.$$

If all investors are uninformed, $q=1$, then

$$p_{22} = \frac{\delta(\lambda_3+\lambda_2)}{\lambda_3+\delta\lambda_2} < 1.$$

If the value of liquidity is very high, $\delta = 0$ (Lizzeri's case), then $p=0$ and ratings are uninformative. ■

Proof of Proposition 12. The results follow from Propositions 7 and 8. ■

Proof of Proposition 13. Consider a market with a high share of informed investors, $q > \overline{q}$ and the rating system described in Proposition 7. Also assume that $\delta > \frac{q\lambda_2}{\lambda_2+\lambda_1}$, and thus unrated types v_2 and v_1 prefer to hold the asset. Now suppose that a regulator imposes a fee cap $\overline{\phi} = \frac{\lambda_3 q(1-\delta)}{\lambda_3 q+\delta\lambda_2}(q\lambda_3 v_3 + \lambda_2 v_2)$ which is less than the fee $(1-\delta)v_3$ charged to type v_3. Then the CRA profit is maximized under the information structure that induces rating two types v_2 and v_3 described in Proposition 8. The regulation is efficient as it allows issue type v_2 to trade. However, the regulation reduces the profits of the CRA. Informed investors gain positive rent. ■

Proof of Proposition 14. The results follow from Propositions 7 and 8. ■

Proof of Proposition 15. The result follows from Proposition 8. ■

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