The value of ex-ante agreements

Anastasia Kartasheva[*]

September 8, 2009

Abstract

This paper analyzes the value of ex-ante agreements that can be renegotiated at later stages. I consider a sequential common agency game between two firms contracting with a common supplier on the adoption of a new technology with unknown costs. After the first firm signs the contract with the supplier, it has an opportunity to delay the execution of the contract. I show that the preliminary agreement between the first firm and the supplier can increase their joint surplus. Also the first firm can reduce the uncertainty at the renegotiation stage by committing to status quo contract favorable to the supplier.

JEL Codes: D82, D86, L51, G38.

[*]The Wharton School, University of Pennsylvania, email: karta@wharton.upenn.edu. I am grateful to Jean-Jacques Laffont and Patrick Rey for encouragement and supervision. I also thank Bruno Jullien, Guido Friebel, David Martimort and Jean Tirole for comments and discussions.

1 Introduction

Decisions are often delayed for information acquisition purposes. Hendricks and Kovenock (1989) study the incentives of firms to delay the drilling decision to explore an oil deposit. The benefit of delay comes from the positive information externality of oil deposit discovery on the probability that the area contains more oil deposits. Rob (1991) develops a model of industry equilibrium in which the delay of entry into the industry comes from the possibility to learn about the profitability of entry by observing past quantities and prices. Benveniste, Busada and Wilhelm (2002) analyze the decision of a start-up to attempt an initial public offering (IPO). If information-production costs regarding performance and prospects are borne primarily by pioneering firms in the industry, the IPO is delayed. As a result, both pioneers and followers remain private and make ill-informed investment decisions. Miller (2005) considers the interaction between firms' decisions to invest in an innovation of uncertain success. He shows that the threat of ex-post entry by a competitor can deter invention ex-ante.

The purpose of the paper is to study the impact of endogenous information acquisition on timing of contract implementation. We consider a relationship between two principals and a single agent under pure information externality. Each principal contracts with the agent for the provision of a good under asymmetric information about the agent's preferences. Once the first principal offers a contract to the agent and the agent agrees to the contract, the outcome of the contract is observed by the second principal. We assume that the first principal can however decide when her contract is to be implemented. To analyze the benefits of postponing implementation, we consider two possibilities. Under *immediate implementation*, the contract signed between the first principal and the agent is implemented before the other principal makes her offer to the agent. Under *postponed implementation*, the first contract is implemented only after the second contract is offered.

The main difference of information structure between the two regimes is that under immediate implementation the contract with the first principal exerts an informational externality on the second contract. If the first contract does not result in full pooling, as will be the case in our analysis, its outcome provides information for the second principal about the agent's preferences. As a result it reduces the agent's informational advantage vis-a-vis the second principal. It in turn implies that the first principal must compensate the agent for any information generated by the implementation of her contract. Therefore, the degree of sensitivity of the first principal's contract to the private information of the agent trades off the benefits of obtaining information about the agent against the cost of revealing this information to the second principal. As a result, in an otherwise a symmetric situation for the two principals, the second principal is always better off. First, she is better informed about the agent's preferences. Second, she has more freedom to make her contract contingent on private information of the agent.

Postponing implementation of the contract to a later stage allows the first principal to avoid leaking any information. If the first principal agrees with the agent on the terms of the contract at the initial stage, but enforces it after the second contracting stage, the information generated by either contract cannot be used by the other principal. For this reason, the first principal is strictly better off by postponing the implementation of her contract[1].

This analysis holds under the assumption that the first principal can commit to the initial contract. To study the robustness of the result, we relax the commitment assumption in Section **??**. We analyze a setting when the first principal can renegotiate the initial contract after the implementation of the second contract. Our analysis suggests that the main result holds even when the first principal can renegotiate the initial offer. Also some new features arise. Anticipating renegotiations, the second principal will adjust her contract to control the information that is revealed to the first principal at the renegotiation stage. At the same time, to reduce the uncertainty about the agent at the renegotiation stage, the first principal may commit to a status quo that is favorable to the agent at renegotiation.

In the literature there are two major explanations why the agents may prefer to delay the actions. First, delay can be used to signal private information. Admati and Perry (1987) analyze a bargaining game in which the time between offers is endogenous and is used to signal the information about the bargainer's strength. They consider a bilateral bargaining model in which the valuation of the buyer is private information. When the prior probability that the buyer has high valuation is sufficiently high, the low valuation buyer delays strategically accepting an offer in order to signal its valuation. When this prior is low, the seller prefers to pool the two types and forgo the surplus it might obtain from a high valuation buyer in order to discourage the low valuation buyer to use delay as a signal.

Second, delay can be employed to acquire information. In addition to research sited above, examples include Pindyck (1991), Chamley and Gale (1994) and Gu and Kuhn (1998, 1999). Pindyck (1991) presents a survey of the literature on irreversible investment and explains that the investment rules can be obtained from methods of option pricing. The investment decisions are largely irreversible, and the investments can be delayed to obtain more information about

[1] Introducing discounting to reflect the cost of delay decreases the benefits of postponed implementation, but does not affect the trade-off between benefits of information acquision and costs of revealing information to the third party.

the market conditions. The irreversible investment opportunity is like a financial call option. The firm can exercise the investment opportunity over a period of time, and it cannot disinvest should the market condition change adversely. As a result delay is beneficial because it allows the firm to be better informed about the value of the investment.

Chamley and Gale (1994) introduce externalities among investors and consider a model in which the payoff of the investor depends on his own action and on the actions of other investors. By delaying the investment decision, the investor becomes better informed about his investment opportunities by observing the decisions of the other investors. In the model with a pure information externality, investors compete for the best place in the decision making queue. Since waiting is costly, the equilibrium of the game is inefficient. If investors expect that some information will be revealed quickly, they strictly prefer to wait and nothing is revealed. The market failure takes one of the two forms. Either there is delay, in which some information is revealed but at a low rate, preventing the impatient investors to benefit from the information. Or, there is an investment collapse in which no information is revealed and there is no investment.

In the context of sequential bargaining between firms and unions in an industry, Kuhn and Gu (1999) show that if characteristics of the firms are correlated, union can acquire information about a firm by delaying the wage negotiations. This learning reduces the disagreements in bargaining and the probability of a strike. Kuhn and Gu find an empirical support for this implication in a panel of Canadian contract negotiations. Gu and Kuhn (1998) rely on a similar idea to explain the continuation negotiations beyond the contract expiry date (holdouts) in labor contract negotiations. In both studies information revelation is always beneficial for the union because it does not have an adverse effect on the profit of the other firm. In contrast, in the present paper the two principals are dealing with the same agent, and the agent suffers from the reduced information advantage. Then the cost of information revelation depends on decision to delay implementation of the contract.

The sited studies highlight the idea that postponing of actions can be beneficial for information acquisition. However, they take as given that the information revelation is advantageous for all the parties. The novel feature of the paper is that we introduce the agency problem into the delay game. The principals do not have any information about the agent's preferences. They are ready to pay for the information to target the contract for a particular type of agent. Due to the agency problem, the information revelation has a cost for the principals. To provide the agent with the appropriate incentives, the first principal has to control how much information is revealed to the second principal through the outcome of her contract. Then postponed implementation is beneficial not only because the delay means better information, but also because it reduces the incentive costs.

We provides an explanation for strategic delay based on a pure information externality. To induce the agent to report its type, the first principal has to compensate the loss of the agent's information advantage vis-a-vis the second principal. The value of the postponed implementation comes from the decreased costs of incentives that the first principal has to inquire to learn the private information of the agent.

The rest of the paper is organized as follows. In the next section we present the model of sequential contracting. In sections 3 and 4 we study the equilibria of the game under immediate and postponed implementation, respectively, and compare the two contracting schedules in section 5. In section 6 we introduce the possibility of renegotiation into the postponed implementation game. Section 7 concludes.

2 The model

Consider a relationship between a supplier S and two firms F_1 and F_2. Each firm F_i, $i = 1, 2$ contracts with a supplier for the provision of an input quantity q_i at price p_i. The two firms use the input to produce the output sold in two different markets. The revenue function $v_i(q_i)$ of F_i is increasing, concave and satisfies the Inada conditions ($v_i' > 0$, $v_i'' < 0$, $v_i'(0) = +\infty$, $v_i'(+\infty) = 0$). The net profit of F_i is

$$V_i = v_i(q_i) - p_i.$$

The supplier produces quantity q at cost θq. The net profit of the supplier dealing with F_i is

$$U_i = p_i - \theta q_i,$$

resulting in the total profit $U = U_1 + U_2$. The supplier accepts contracts that guarantee non-negative profit.

The cost parameter θ is supplier's private information. It can take two values, θ_L and θ_H, $\Delta\theta \equiv \theta_H - \theta_L > 0$, with probabilities ν and $1 - \nu$ respectively. The distribution of θ is common knowledge.

A contract C_i between F_i and S is a communication mechanism that consists of a message space M_i and a measurable mapping $\gamma_i : M_i \longmapsto \Delta(\mathbb{R}_+) \times \mathbb{R}$, where $\delta_i(m_i) \in \Delta(\mathbb{R}_+)$ and $p_i(m_i) \in \mathbb{R}$ stand for the lottery over the input quantity and the expected price associated with message $m_i \in M_i$. $\delta_i(\, q_i |\, m_i)$ will denote the conditional probability of q_i given m_i. The supplier's reporting strategy in the machanism (γ_i, M_i) is a mapping $\sigma_i : \{\theta_L, \theta_H\} \rightarrow M_i$. I follow the literature on common agency games (e.g. Martimort (1996)) and assume that the firm F_i is unable to contingent her contract on the contract of the supplier with the other firm F_j.

The contracts (mechanisms) and outcomes $(p_i, q_i)_{i=1,2}$ are public. Each firm F_i observes the mapping $\gamma_j : M_j \longmapsto \Delta(\mathbb{R}_+) \times \mathbb{R}$, the price p_j and the output $v_j(q_j)$ of F_j. The communication between F_i and S is private. The message m_i reported by S to F_i is not observed by F_j.

The firms contract with the supplier sequentially at two different dates.

Within the context of this model, I derive the value of contract implementation delay. To address the question, I consider two games. Under *immediate contract implemetation*, F_i and S sign and immediately execute the contract at date i. Under *postponed contract implementation*, F_1 and S sign the contract at date 1, but the contract execution is delayed till date 3, after F_2 and S sign and execute their contract. At date 3, F_1 and S have a possibility to renegotiate the contract.

The timing of the immediate implementation game is as follows.

Date 0. The supplier learns his type θ.

Date 1. The firm F_1 offers a contract C_1. The supplier accepts or rejects the contract. An accepted contract is executed.

Date 2. The firm F_2 offers a contract C_2. The supplier accepts or rejects the contract. An accepted contract is executed.

Under postponed contract implementation, the timing incorporates the delay of contract execution and the possibility of renegotiation by F_1. The game unfolds as follows.

Date 0. The supplier learns his type θ.

Date 1. The firm F_1 offers a contract C_1. The supplier accepts or rejects the contract. The execution of the contract is postponed till date 3.

Date 2. The firm F_2 offers a contract C_2. The supplier accepts or rejects the contract. An accepted contract is executed.

Date 3. The firm F_1 has an option to renegotiate the original contract C_1 and offer a new contract C_1^R. If the supplier agrees to the terms of the new contract, this contract is executed. Otherwise, the original contract C_1 is executed.

In the following I analyze the Perfect Bayesian equilibria of the immediate and posponed contract implementation games.

The complete information benchmark. Suppose the firms know the supplier's type θ. Then each firm offers a contract $p_i = \theta q_i$. F_i demands the quantity that maximizes the joint surplus,

$$V_i + U_i = v_i(q_i) - \theta q_i.$$

The optimal quantity $q_i^{FI}(\theta)$ is implicitly defined by $v_i'(q_i) = \theta$. Under this contract, the supplier earns zero profit with each firm, $U_i = 0$.

3 Immediate Contract Implementation

The execution of the contract of the first firm F_1 may provide new information about the supplier's type to the second firm F_2. The main result of this section is that the information externality is harmful to the first firm. The reason is that F_1 has to compensate the supplier for the loss of information advantage vis-a-vis F_2. To decrease the cost of information revelation, F_1 reduces the sensitivity of her contract to supplier's private information. At the same time, the second firm F_2 is better informed and thus benefits from the information externality.

The characterization of optimal contracts under immediate contract implementation is closely related to the the results of the literature on renegotiation of dynamic contracts. Hart and Tirole (1988), Dewatripont (1989) and Laffont and Tirole (1990) have analyzed the multiperiod interaction between a principal and an agent under asymmetric information. They have found that the ability of the parties to renegotiate the allocative inefficiencies of the initial contract ex post weakens the principals ability to screen the agent's private information and lowers the ex ante joint surplus. The optimal initial contract involves partial learning of the agent's type. Imperfect learning is implemented by the means of a stochastic contract such that the agent randomizes between several contract choices. Bester and Strausz (2001) provide the general characterization result that extends the Revelation Prinicple to dynamic principal-agent games under imperfect commitment.

The optimal contract of F_1 often involves partial learning of the supplier's type. The extend of pooling depends on the value of private information to the supplier. The higher are the potential profits of the supplier with the second firm, the lower is the ability of F_1 to compensate the supplier for the information to make her contract contigent on supplier's type. Calzolari and Pavan (2006) study an information exchange between two principals who contract siquentially with the same agent. Had F_1 the opportunity to decide whether F_2 observes the outcome of transaction with the supplier, the results of Calzolari and Pavan imply that no information disclosure would have occured.

In the following I characterize the optimal contracts of F_1 and F_2. The game is solved backwards. First, I derive the optimal contract of F_2 for arbitrary beliefs about the supplier's type. Then I analyze the optimal stochastic contract of F_1 and discuss the cost of immediate contract execution.

3.1 Optimal contract C_2

The contract C_1 and the outcome (p_1, q_1) may change the beliefs of F_2 about the costs of the supplier. When the transaction between F_1 and S perfectly reveals the type of the supplier, F_2 and S sign a contract under complete information.

Now consider a situation when F_2 is uncertain about the type θ. Denote by $\mu \in (0,1)$ the belief of F_2 that the supplier has low cost θ_L. The optimal contract C_2 is a menu of two profiles (p_{2t}, q_{2t}), $t \in \{L, H\}$ that are incentive compatible,

$$p_{2t} - \theta_t q_{2t} \geq p_{2j} - \theta_t q_{2j},\ \ t, j \in \{L, H\}$$

satisfy the participation constraint of the supplier,

$$p_{2t} - \theta_t q_{2t} \geq 0,\ \ t \in \{L, H\}$$

and maximize the expected profit of F_2,

$$\max_{(t_2, q_2)} \mu(v_2(q_{2L}) - p_{2L}) + (1-\mu)(v_2(q_{2H}) - p_{2H}).$$

In this problem, a low cost type gains positive rent $U_{2L} = \Delta\theta q_{2H}$ by overstating the cost and selecting the profile desinged for type H. To limit the rent paid to this type, F_2 reduces the input quantity purchased from a high cost supplier. A standard rent extraction versus efficiency trade-off results in a downward distortion of high cost type quantity and a positive rent of the low cost type.

The next proposition summarizes the properties of the contract C_2.

Proposition 1 *When F_2 is perfectly informed about the cost of the supplier, $\mu \in \{0, 1\}$, the contract C_2 is a full information contract. The supplier type i produces the efficient quantity q_{2i}^{FI} and obtains zero profits.*

When F_2 is uncertain about the cost of the supplier, $\mu \in (0,1)$, the contract C_2 is a second-best contract. The high cost supplier's quantity q_{2H} is distorted downward,

$$v_2'(q_{2H}) = \theta_H + \frac{\mu}{1-\mu}\Delta\theta.$$

As the belief μ increases, the downward distortion of q_{2H} becomes more pronounced. This supplier gains zero profits. The low cost supplier produces the efficient quantity q_{2L}^{FI} and obtains positive profits $U_{2L} = \Delta\theta q_{2H}$.

When contracting with F_2, the ability of the θ_L type supplier to obtain the rent depends on the information created by the contract C_1. As the outcome of the transaction of the supplier with F_1 becomes more informative about the supplier's type, F_1 must pay higher price to compensate the supplier for the loss of information advantage vis-a-vis the other firm F_2.

3.2 Optimal contract C_1

An optimal contract C_1 is a menu of two lotteries designed for each type of the supplier. To illustrate the benefits of a stochastic contract, consider an optimal deterministic contract C_1^d which restricts F_1 to offer a choice between two incentive compatible outcomes, (p_{1L}^d, q_{1L}^d) and (p_{1H}^d, q_{1H}^d). Then the outcome of C_1^d reveals the supplier's type to F_2. The incentive compatibility constraints of suppliers θ_L and θ_H are

$$\begin{aligned} p_{1L}^d - \theta_L q_{1L}^d &\geq p_{1H}^d - \theta_L q_{1H}^d + \Delta\theta q_{2H}^{FI}, \\ p_{1H}^d - \theta_H q_{1H}^d &\geq p_{1L}^d - \theta_H q_{1L}^d + \max\{0, U_{2L}(p_{1L}^d, q_{1L}^d)\}. \end{aligned}$$

When the suppliers honestly report their types to F_1, both types obtain zero profits with F_2. However, the effect of misreporting is different for the two types. The high cost supplier gains negative profits when F_2 beleifs that supplier's cost is θ_L, $U_{2L}(p_{1L}^d, q_{1L}^d) = \theta_L q_{2L}^{FI} - \theta_H q_{2L}^{FI} < 0$. Thus, following the misreporting of his type, he will refuse the contract C_2 and obtain zero profit with F_2. On the contrary, the low cost supplier gains when F_2 beliefs that he has high cost. The rent $\Delta\theta q_{2H}^{FI}$ is proportional to the quantity that F_2 purchases from the type θ_H. Concequently, to induce honest reporting, F_1 must pay this rent to the supplier.

The rent $\Delta\theta q_{2H}^{FI}$ could have been lower were F_2 uncertain that she is facing the θ_H type. In that case, the contract C_2 entails a downward distortion of the output q_{2H} and thus reduces the rent. A lottery where the price and quantity profile designed for the high cost supplier is offered to the low cost supplier with a positive probability achieves this objective. The next Lemma discribes the structure of the optimal lottery.

Lemma 1 *The optimal contract C_1 has the following structure: There are two outcomes (p_{1L}, q_{1L}) and (p_{1H}, q_{1H}). The θ_L type supplier obtains the outcome (p_{1L}, q_{1L}) with probability σ and the outcome (p_{1H}, q_{1H}) with probability $1-\sigma$. The θ_H type supplier obtains the outcome (p_{1H}, q_{1H}) with certainty.*

When F_2 observes the contract (p_{1L}, q_{1L}), she knows with certainty that the supplier has low cost. In this case F_2 contracts with the supplier under complete information, $\mu = 1$. When F_2 observes the contract (p_{1H}, q_{1H}), she is uncertain about the supplier's type because both types obtain this outcome with a positive probability. F_2 updates her beliefs according to the Bayes rule,

$$\mu = \frac{(1-\sigma)\nu}{(1-\sigma)\nu + 1 - \nu},$$

and offers a contract C_2 under beliefs μ. In this case the posterior probability that the supplier type is θ_L is lower than the prior, $\mu < \nu$.

Of course, this strategy has a cost for F_1. Designing a stochastic contract implies that the deal between F_1 and S becomes less sensitive to the supplier's costs. Hence, an optimal lottery trade-offs the benefits of reducing the price paid to the low cost supplier and the costs of reducing the expected quantity purchased from this supplier.

An optimal stochastic contract under immediate implementation maximizes the expected profit of F_1 under the incentive and participation constraints of the two supplier types,

$$\max_{(\sigma, p_1, q_1)} \nu[\sigma(v_1(q_{1L}) - p_{1L}) + (1-\sigma)(v_1(q_{1H}) - p_{1H})] + (1-\nu)(v_1(q_{1H}) - p_{1H})$$
$$IC_L: \quad \sigma(p_{1L} - \theta_L q_{1L}) + (1-\sigma)(p_{1H} - \theta_L q_{1H} + \Delta\theta q_{2H}) \geq p_{1H} - \theta_L q_{1H} + \Delta\theta q_{2H},$$
$$IC_H: \quad p_{1H} - \theta_H q_{1H} \geq p_{1L} - \theta_H q_{1L},$$
$$PC_L: \quad p_{1L} - \theta_L q_{1L} \geq 0 \text{ and } p_{1H} - \theta_L q_{1H} + \Delta\theta q_{2H} \geq 0,$$
$$PC_H: \quad p_{1H} - \theta_H q_{1H} \geq 0.$$

In this program, constraint PC_L states that both outcomes must guarantee non-negative profits to a low cost supplier. The next proposition summarizes the properties of the optimal stochastic contract C_1.

Proposition 2 *Under immediate implementation, the optimal contract of F_1 consists of two profiles, (p_{1L}, q_{1L}) and (p_{1H}, q_{1H}) and a mixing strategy σ. The profile (p_{1L}, q_{1L}) is offered to the low cost supplier with probability σ. It entails the efficient quantity $q_{1L} = q_{1L}^{FI}$ and the price $p_{1L} = \theta_L q_{1L}^{FI} + \Delta\theta(q_{1H} + q_{2H})$. The profile (p_{1H}, q_{1H}) is offered to the low cost supplier with probability $1-\sigma$ and the high cost supplier with probability one. It entails the downward distortion of quantity q_{1H},*

$$v_1'(q_{1H}) = \theta_H + \frac{\nu\sigma}{1-\nu\sigma},$$

and the price $p_{1H} = \theta_H q_{1H}$. The low cost supplier obtains positive profits $U_1 = \Delta\theta(q_{1H} + q_{2H})$, while the high cost supplier obtains zero profits. The optimal mixing strategy σ is implicitely defined by

$$[v_1(q_{1L}^{FI}) - \theta_L q_{1L}^{FI} - \Delta\theta q_{1H} - \Delta\theta q_{2H})] - [v_1(q_{1H}) - \theta_H q_{1H}] = \sigma\Delta\theta\frac{dq_{2H}}{d\sigma}.$$

When $\sigma = 0$, the optimal contract of F_1 is a pooling contract that entails the same quantity and price for both supplier types. When $\sigma = 1$, the optimal contract is fully separating.

The optimal mixing strategy σ is determined by the two couterveiling effects of offering the profile (p_{1H}, q_{1H}) to the low cost supplier on the payoff of F_1. On one hand, offering this profile reduces the profits of F_1 dealing with the low cost supplier. The size of the effect is equal to the difference between the surplus created by the low cost supplier, $v_1(q_{1L}^{FI}) - \theta_L q_{1L}^{FI} - \Delta\theta q_{1H} - \Delta\theta q_{2H})$, and the surplus created by the high cost supplier, $v_1(q_{1H}) - \theta_H q_{1H}$. On the other hand, mixing reduces the rent that F_1 pays to the supplier under profile (p_{1L}, q_{1L}) because it reduces the information revealed to F_2.

Two special cases occur when there is no mixing, $\sigma = 0$ or $\sigma = 1$. When $\sigma = 0$, F_1 offers the same quantity and output profile to both types of suppliers. This outcome occurs when the costs of information revelation exceed the gains of screening suppliers. For example, the second firm can be substantially larger than the first one, $v_2(q) > v_1(q)$. Then the cost of information rent that the first firm has to pay to learn the supplier's type can make screening too costly, and the pooling obtains. On the contrary, $\sigma = 1$ corresponds to the situation when the second firm is smaller than the first one, and thus the rent that F_1 has to pay is negligible compared to the benefits of targeting the offers to the supplier type.

The mixing $\sigma \in (0, 1)$ has two countervailing effects on the quantity sold by the high cost supplier. On one hand, it decreases the quantity purchased by F_2 because the contract C_2 is signed under asummetric information. On the other hand, it enables F_1 to purchase more input than under the deterministic contract. As F_1 pays the rent to the low cost supplier with probability $\sigma\nu < \nu$, she can reduce the quantity distortion compared to the deterministic contract.

Finally, if the two firms are symmetric, the next result obtains.

Corollary 1 *When the two firms are symmetric, $v_1(q) = v_2(q)$, F_2 obtains higher expected payoff then F_1.*

The second firm has two information advantages. First, it is weakly better informed than the first firm. Second, it does not need to compensate the supplier for the information when the contract revelas supplier's type.

4 Preliminary Agreement

The results of the previous section suggest that delaying the transaction with the supplier reduces the cost of information acquisition and increases the profits of the firm. In this section I derive the optimal preliminary agreement that F_1 can sign with the supplier before the transaction of F_2 and renegotiate it at the later stage.

4.1 Optimal renegotiated contract C_1^R

The transaction between F_2 and S can open possibilities for F_1 to improve the original contract C_1. Naturally, offers that reduce the rent of the supplier will not be accepted. However, F_2 can improve his payoff by renegotiating the quantity. If the outcome of C_1 perfectly reveals the supplier's costs, the renegotiated quantities are at the efficient level, q_{1i}^{FI}, $i = L, H$. Under asymmetric information, F_1 holds beliefs μ and offers quantity–price profiles (p_{1i}^R, q_{1i}^R) that are incentive compatible,

$$p_{1i}^R - \theta_i q_{1i}^R \geq p_{1j}^R - \theta_i q_{1j}^R,\ i, j = L, H,$$

guarantee the rent U_{1i}, $i = L, H$ of the original contract C_1 to both supplier types,

$$p_{1i}^R - \theta_i q_{1i}^R \geq U_{1i}, \tag{1}$$

and maximize the expected profit of F_1,

$$\max_{(p_1^R, q_1^R)} \mu(v_1(q_{1L}^R) - p_{1L}^R) + (1 - \mu)(v_1(q_{1H}^R) - p_{1H}^R).$$

Normalizing the rent of the high cost supplier to zero, $U_{1H} = 0$, the optimal renegotiated contract C_1^R is one of the the following three kinds. I follow the terminology of Laffont and Tirole to name the contracts.

I. Conditionally optimal contract. If the rent U_{1L} is sufficiently low and thus the participation constraint (1) is not binding, the renegotiated contract will increase the output purchased from the high cost supplier and the rent of the low cost supplier. In this case, the incentive constraint of type θ_L and the participation constraint of type θ_H are the binding. The optimal contract C_1^R entails the efficient quantity of the low cost supplier, q_{1L}^{FI}, and a downward distortion of the quantity of the high cost supplier,

$$v_1'(q_{1H}^R) = \theta_H + \frac{\mu}{1-\mu}\Delta\theta.$$

The prices paid for these quantities are $p_{1L}^R = \theta_L q_{1L}^{FI} + \Delta\theta q_{1H}^R$ and $p_{1H}^R = \theta_H q_{1H}^R$. Consequently, the rents of the low and high cost suppliers are $U_{1L}^R = \Delta\theta q_{1H}^R$ and $U_{1H}^R = 0$ respectively. This contract is optimal when $U_{1L} \leq \Delta\theta q_{1H}^R$.

II. Rent constrained contract. If the rent of the efficient type is higher than $\Delta\theta q_{1H}^R$, F_1 is limited by the participation constraint of the low cost supplier to increase the quantity purchased from the high cost supplier. In this case the renegotiated contract is detemined by the incentive and participation constraints of the low cost supplier. This supplier selles the efficient quantity and obtains the same rent as under the original contract, U_{1L}. The quantity of the high cost supplier is increased to

$$q_{1H}^R = \frac{U_{1L}}{\Delta\theta}$$

and her rent is zero, $U_{1H} = 0$. The rent constrainted contract is optimal when $\Delta\theta q_{1H}^R \leq U_{1L} \leq \Delta\theta q_{1H}^{FI}$.

III. Sell-out contract. If the rent under the original contract is higher than $\Delta\theta q_{1H}^{FI}$, the participation constraint of the low cost type is binding, resulting in the contract that entails efficient quantities for both supplier types and the original rent profile U_{1i}, $i = L, H$. The sell out contract is optimal when $\Delta\theta q_{1H}^{FI} \leq U_{1L} \leq \Delta\theta q_{1L}^{FI}$.

4.2 Optimal contract C_2

The possibility of contract renegotiation between F_1 and S changes the cost of information for F_2. Like F_1 in immediate contract implementaiton game, F_2 may decide to make her contract less sensitive to the supplier's private information in order to reduce the rent paid to the low cost type. The difference between the current contract and the contract of F_1 under immediate implementation is that now the supplier has the default option U_{1i}, $i = L, H$ defined by the original contract C_1. If this contract guarantees some positive rent to the supplier, the cost of information to the second firm is reduced. Indeed, now the supplier is concerned about revealing his private information only to the extend that this precludes earning rents above those specified in the original contract C_1.

When F_2 offers a stochastic contract to the low cost supplier, the structure of the lottery is the same as the lottery of F_1 under immediate implementation. F_2 designs two profiles

(p_{2L}, q_{2L}) and (p_{2H}, q_{2H}). The low cost supplier obtains profile (p_{2L}, q_{2L}) with probability σ_2 and profile (p_{2H}, q_{2H}) with probability $1 - \sigma_2$. The high cost supplier obtains profile (p_{2H}, q_{2H}) with probability one.

The outcome (p_{2L}, q_{2L}) reveals that the supplier has low cost, $\mu = 1$. Unless the original contract C_1 entails the inefficient quantity of the low cost supplier, there is no room for renegotiation. The outcome (p_{2H}, q_{2H}) results in beliefs updating

$$\mu = \frac{\nu(1 - \sigma_2)}{\nu(1 - \sigma_2) + 1 - \nu}.$$

Renegotiation can be mutually beneficial for F_1 and S when the high cost supplier quantity of the original contract is lower than what can be acieved under beliefs μ.

The supplier incentive constraints must reflect the renegotiation stage. Following the outcome (p_{2L}, q_{2L}), the low cost supplier obtains the rent of the original contract U_{1L}. Following the other outcome (p_{2H}, q_{2H}), the renegotiation can increase his payoff resulting in the rent $\max\{U_{1L}, \Delta\theta q_{1L}^R\}$. Similarly, should the high cost supplier decide to understate his cost, the outcomes (p_{2L}, q_{2L}) and (p_{2H}, q_{2H}) will result in payoffs $\max\{0, U_{1L} - \Delta\theta q_{1L}\}$ and zero, respectively. Thus the incentive costraints are

$$\begin{aligned} IC_{2L} \quad &: \quad \sigma_2(p_{2L} - \theta_L q_{2L} + U_{1L}) + (1 - \sigma_2)(p_{2H} - \theta_L q_{2H} + \max\{U_{1L}, \Delta\theta q_{1L}^R\}) \\ &\geq \quad p_{2H} - \theta_L q_{2H} + \max\{U_{1L}, \Delta\theta q_{1L}^R\}, \end{aligned} \tag{2}$$

$$IC_{2H}: \quad p_{2H} - \theta_H q_{2H} \geq \sigma_2(p_{2L} - \theta_H q_{2L} + \max\{0, U_{1L} - \Delta\theta q_{1L}\}) + (1 - \sigma_2)(p_{2H} - \theta_H q_{2H}). \tag{3}$$

The participation constraints are

$$p_{2i} - \theta_i q_{2i} \geq 0, \; i = L, H. \tag{4}$$

The optimal contract C_2 maximizes the expected profit of F_2,

$$\max_{(p_2, q_2, \sigma_2)} \nu\sigma(v_2(q_{2L}) - p_{2L}) + (1 - \nu\sigma)(v_2(q_{2H}) - p_{2H})$$

subject to constraints (2), (3) and (4). The next proposition summarizes the properties of the optimal contract.

Proposition 3 *Under postponed contract implementation, the optimal contract of F_2 consists of two profiles, (p_{2L}, q_{2L}) and (p_{2H}, q_{2H}) and a mixing strategy σ_2. The profile (p_{2L}, q_{2L}) is* *offered to the low cost supplier with probability σ_2. It entails the efficient quantity $q_{2L} = q_{2L}^{FI}$ and the price $p_{2L} = \theta_L q_{2L}^{FI} + \Delta\theta q_{2H} + \max\{\Delta\theta q_{1L}^R - U_{1L}, 0)$. The profile (p_{2H}, q_{2H}) is offered to the low cost supplier with probability $1 - \sigma_2$ and the high cost supplier with probability one. It entails the downward distortion of quantity q_{2H},*

$$v_1'(q_{2H}^R) = \theta_H + \frac{\nu\sigma_2}{1 - \nu\sigma_2},$$

and the price $p_{2H} = \theta_H q_{2H}$. The low cost supplier obtains positive profits, $U_{2L} = \Delta\theta q_{1H} + \max\{\Delta\theta q_{1L}^R - U_{1L}, 0)$, while the high cost supplier obtains zero profits, $U_{2H} = 0$. The optimal mixing strategy σ_2 is implicitely defined by

$$[v_2(q_{2L}^{FI}) - \theta_L q_{2L}^{FI} - \Delta\theta q_{2H} - \max\{\Delta\theta q_{1L}^R - U_{1L}, 0)] - [v_1(q_{1H}) - \theta_H q_{1H}] = I(\Delta\theta q_{1L}^R - U_{1L})\sigma_2 \Delta\theta \frac{dq_{1H}^R}{d\sigma_2},$$

where $I(\Delta\theta q_{1L}^R - U_{1L})$ is the indicator function equal to one when $\Delta\theta q_{1L}^R > U_{1L}$ and zero otherwise. When $\sigma = 0$, the optimal contract of F_1 is a pooling contract that entails the same quantity and price for both supplier types. When $\sigma = 1$, the optimal contract is fully separating. The optimal mixing probability σ_2 is increasing in the rent U_{1L} that the initial contract C_1 guarantees the low cost supplier, i.e. $\frac{d\sigma_2}{dU_{1L}} > 0$.

The main difference between the contract C_2 under postponed implementation and the contract C_1 under immediate implementation is that now the contract of F_2 depends on the rent that is guaranteed to the supplier by the initial contract with F_1. As a result, the mixing strategy σ_2 depends on the rent U_{1L} of the initial contract C_1. The higher is the rent U_{1L} guaranteed by the original contract, the less the supplier is concerned that revealing his type to F_2 diminishes the ability to earn positive rent in the future. An important implication of this result is that by changing the amount of rent offered to the supplier in the original contract C_1, F_1 can control the information that is avaialble at the renegotiation stage.

4.3 Optimal preliminary contract $\mathbf{C}_1$

What is the rent that the optimal preliminary contract of F_1 guarantees to the low cost supplier? From the perspective of date zero, the optimal contract C_1 maximizes the expected payoff of F_1 anticipating the contract of F_2 and the possible renegotiation outcome,

$$\begin{aligned}
&\max_{U_{1i}, q_{1i}} \nu\sigma_2(U_{1L})(v_1(q_{1L}) - \theta_L q_{1L} - U_{1L}) \\
&+\nu(1 - \sigma_2(U_{1L}))(v_1(q_{1L}) - \theta_L q_{1L} - \max\{\Delta\theta q_{1H}^R(\sigma(U_{1L})), U_{1L}\}) \\
&+(1 - \nu)\max\{(v_1(q_{1H}) - \theta_H q_{1H}), (v_1(q_{1H}^R) - \theta_H q_{1H}^R)\}.
\end{aligned}$$

The expected payoff of F_1 is composed of three terms. With probability $\nu\sigma_2(U_{1L})$ the outcome of the contract of F_2 reveals the type of the low cost supplier to F_1. In this case the low cost supplier produces the efficient quantity q_{1L}^{FI} and gains the profits U_{1L}. With probability $v(1-\sigma_2(U_{1L}))$ F_1 deals with the low cost supplier under asymmetric information. The supplier produces the efficient quantity q_{1L}^{FI}. The supplier's profits in this case depend on whether there is scope for renegotiation of the initial contract C_1. If no renegotiation occurs, S gains U_{1L}. Following the renegotiations, the profits are $\max\{\Delta\theta q_{1H}^R(\sigma(U_{1L})), U_{1L}\}$. Finally, with probability $1-\nu$, F_1 deals with a low cost supplier. The supplier obtains zero profits and produces either the quantity q_{1H} of the initial contract C_1 or the quantity q_{1H}^R of the renegotiated contract.

The contract C_1 must be incentive compatible,

$$\begin{aligned} U_{1L} &\geq U_{1H} + \Delta\theta q_{1H}, \\ U_{1H} &\geq U_{1L} - \Delta\theta q_{1L}, \end{aligned}$$

and satisfy the supplier's participation constraint, for $i = L, H$,

$$U_{1i} \geq 0.$$

The rent U_{1L} offered to the low cost supplier has two effects on the expected payoff of F_1. On one hand, it imposes direct costs on F_1. On the other hand, it makes the contract of F_2 more informative about the supplier's type by reducing the probability of pooling, $1-\sigma(U_{1L})$ (Proposition 3). Thus it lowers the downward distortion of the quantity purchased from the high cost supplier.

One feasible contract of F_1 is a null contract. Under this contract, F_1 offers zero quantity and price to both supplier types, and F_1 and S agree to contract at date 3. Naturally, this contract is always accepted by the supplier and is renegotiated to postive quantities and prices at date 3. Effectively, the null contract changes the sequence of contracting between F_1 and F_2, and Corollary 1 suggests that F_1 can increase her expected payoff by the delay.

The payoff of F_1 in the first stage writes

$$\begin{aligned} V_1^R &= \nu\sigma(\frac{1}{2}\overline{\theta}^2 - \overline{u}_1^0) + (1-\nu\sigma)[\frac{\nu(1-\sigma)}{1-\nu\sigma}(\frac{1}{2}\overline{\theta}^2 - \Delta\theta(\underline{\theta} - \Delta\theta\frac{\nu(1-\sigma)}{1-\nu})) \\ &\quad + \frac{1-\nu}{1-\nu\sigma}(\underline{\theta}\underline{q}_1^R - \frac{1}{2}(\underline{q}_1^R)^2) \end{aligned}$$

With probability $\nu\sigma$ P_1 faces a high valuation agent and becomes fully informed about his type at the renegotiation stage. In this case she pays the rent $\overline{u}_1^0$ to the agent. With probability $1-\nu\sigma$ P_1 renegotiates a contract under asymmetric information. She updates the beliefs according to the Bayes rule. If P_1 faces a high valuation agent , the rent left to the agent equals to $\Delta\theta \underline{q}_1^R$. If P_1 faces the low valuation agent, he is left with no rent and is offered the output $\underline{q}_1^R$.

Deriving V_1^R with respect to $\overline{u}_1^0$ and evaluating it at $\overline{u}_1^0 \to 0$, we obtain the following result.

Proposition 4 *When P_1 increases the rent promised to the high valuation agent at the first stage, its net impact on her expected payoff is equal to*

$$\left.\frac{dV_1^R}{d\overline{u}_1^0}\right|_{\overline{u}_1^0 \to 0} = \nu \frac{d\sigma}{d\overline{u}_1^0}(\Delta\theta(\underline{\theta} - \Delta\theta \frac{\nu(1-\sigma)}{1-\nu}) - \overline{u}_1^0) - \nu\sigma.$$

It is composed of two countervailing effects. It has a positive effect because P_1 becomes better informed about the agent at the renegotiation stage, and it has a negative effect because P_1 commits to pay the rent $\overline{u}_1^0$ even when she is perfectly informed about the agent's type.

When P_1 increases $\overline{u}_1^0$, it has two countervailing effects on her payoff. First, it increases the payoff of P_1 because by promising some rent to the high valuation agent, P_1 induce P_2 to offer a "more separating". As a result, P_1 becomes better informed about the agent at the renegotiation stage, and can reduce the rent that has to be left to the agent at that stage. The second effect is negative, and it is due to the fact that by promising some positive rent to the agent, P_1 commits to pay this rent even when she becomes perfectly informed about the agent's valuation, with probability σ. The overall impact of $\overline{u}_1^0$ depends on the relative magnitude of each of these effects.

5 Conclusion

In this paper we analyzed the incentives of one principal to postpone the implementation of her contract when she anticipates that the agent will contract with the other principle in the future. We has shown that when the first principal can commit to the contract, she is better off under postponed implementation. The benefit comes from the reduced cost of incentives she has to provide to the agent to report his private information. When the agent contracts with the first principal, he is concerned with the information that the outcome of the contract reveals to the second principal. Thus, to induce the agent to report his private information, the first principal has to compensate him for the loss of information advantage vis-a-vis the second one. By postponing the implementation of the contract, the first principal avoid the information leakage produced by the outcome of her contract. This result implies that is always better to be a follower in the decision queue: First, the second principal is better informed about the agent's preferences. Second, she has more freedom to target her contract to a particular type of agent.

We study the robustness of the result to the commitment assumption. When the first principal can postpone the implementation of her contract, by the time the initial contract is to be implemented there is new information that becomes available after the implementation of the contract between the second principal and the agent. Naturally, if the initial contract is ex-post inefficient, the first principal and the agent can gain from renegotiating the contract. The possibility of the renegotiation affects the optimal contract of the second principal who designs a contract to control the information available to the first principal at the renegotiation stage. We characterize the outcome of the renegotiation stage and the optimal contract of the second principal. Also we show that the first principal may have incentives to offer some positive rent to the agent at the first stage to become better informed about the agent at the renegotiation stage.

Appendix

Proof of Proposition 1. The proof is standard and it is omitted[2].

Let us first consider a relaxed problem in which we ignore $\underline{IC}_2^R$ and $\overline{PC}_2^R$. As it is often the case, these two constraints are not binding, and we can verify ex-post that they are indeed satisfied for the solution of the problem C_2^R. ■

In the next lemma we show that rent - constrained contract does not arise in equilibrium.

Lemma 2 *In equilibrium, it is necessary that* $\overline{u}_1^0 \leq \Delta\theta \underline{q}_1^R$.

Proof. Suppose that $\overline{u}_1^0 > \Delta\theta \underline{q}_1^R$. Then the program C_2^R is independent of $\underline{q}_1^R$. It implies that it is optimal for P_2 to set $\sigma = 1$. Hence, observing the outcome of C_2 provides P_1 with full information about the agent's type at the renegotiation stage. Then, by understating his valuation to P_2, the high valuation agent can obtain the rent $\Delta\theta\underline{\theta}$, and it must be that $\overline{u}_1^0 > \Delta\theta\underline{\theta}$. However, $\Delta\theta\underline{\theta}$ is the highest rent that the agent can obtain by manipulating its report, and the initial choice of $\overline{u}_1^0$ may not be optimal. A contradiction. ■

Lemma 2 implies that the optimal renegotiated contract must be conditionally optimal contract of Case 1. Given that $\overline{u}_1^0 \leq \Delta\theta \underline{q}_1^R$, the next proposition describes the optimal contract of P_2.

Proof of Lemma 1.

[2] The optimal contract C_2^A is such that P_2 contracts with both types. It is the case when, under prior beliefs ν, the expected gain of offering a contract to $\underline{\theta}$ is higher than the expected rent rewarded to $\overline{\theta}$, that is $\frac{1}{2}(1-\nu)\underline{\theta}^2 - \nu\Delta\theta(\underline{\theta} - \frac{1}{2}\Delta\theta\frac{\nu}{1-\nu}) > 0$. The optimal contract C_1 is such that $\mu < \nu$, and this condition is sufficient to guarantee that P_2 indeed offers a contract to both types.

From Kartasheva (2004) an optimal contract of P_1 is a pair of lotteries designed for each type of agent. To save on notations, denote $x=(t,q)$ an element of the support, and $\overline{X}=\{\overline{x}_1,...,\overline{x}_k\}$ and $\underline{X}=\{\underline{x}_1,...,x_m\}$ the support of the lottery of each type of agent, with $k,m<+\infty$. Then the lottery assigns the distribution $\pi(\theta)=(\pi_1(\theta),...,\pi_{\#(X(\theta))})$, where $0\leq\pi_i(\theta)\leq 1$ and $\sum\limits_{i=1}^{\#(X(\theta))}\pi_i(\theta)=1$. The optimal contract of P_1 is a choice of two lotteries $(\overline{\pi},\overline{X})$ and $(\underline{\pi},\underline{X})$ that solves the next problem

$$\begin{array}{ll}\max\limits_{(\pi,X)} & \nu\sum\limits_{i=1}^{k}\overline{\pi}_i(\overline{t}_i-\frac{1}{2}\overline{q}_i^2)+(1-\nu)\sum\limits_{i=1}^{m}\underline{\pi}_i(\underline{t}_i-\frac{1}{2}\underline{q}_i^2)\\ \overline{IC}_1: & \sum\limits_{i=1}^{k}\overline{\pi}_i(-\overline{t}_i+\overline{\theta}\overline{q}_i+\max\{\overline{u}_2(\overline{t}_i,\overline{q}_i),0\})\geq\sum\limits_{i=1}^{m}\underline{\pi}_i(-\underline{t}_i+\overline{\theta}\underline{q}_i+\max\{\overline{u}_2(\underline{t}_i,\underline{q}_i),0\}),\\ \underline{IC}_1: & \sum\limits_{i=1}^{m}\underline{\pi}_i(-\underline{t}_i+\underline{\theta}\underline{q}_i)\geq\sum\limits_{i=1}^{k}\overline{\pi}_i(-\overline{t}_i+\underline{\theta}\overline{q}_i+\max(\underline{u}_2(\overline{t}_i,\overline{q}_i))),\\ \overline{PC}_1: & -\overline{t}_i+\overline{\theta}\overline{q}_i\geq 0\text{ for all }i=1,...,k,\\ \underline{PC}_1: & -\underline{t}_i+\underline{\theta}\underline{q}_i\geq 0\text{ for all }i=1,...,m.\end{array}$$

Claim 1. *For the contract C_1 to leave any uncertainty about the agent's private information, it must be that $\overline{X}\cap\underline{X}\neq\emptyset$.*

Proof. Suppose it is not the case, and $\overline{X}\cap\underline{X}=\emptyset$. Then upon observing any outcome x of the contract, the posterior belief of P_2 that the agent has high valuation is

$$\mu=\begin{cases}1, \text{ if } x\in\overline{X},\\ 0, \text{ if } x\in\underline{X}.\end{cases}$$

A contradiction.

Denote $X^c=\overline{X}\cap\underline{X}$ the elements that are common for the support of both lotteries. Then $\overline{X}\setminus X^c$ and $\underline{X}\setminus X^c$ contain the elements that are unique for the support of each lottery.

Claim 2. *Suppose that $\#(X(\theta)\setminus X^c)\geq 2$. Then P_1 is better off by offering a lottery with $\#(X(\theta)\setminus X^c)=1$.*

Proof. Consider the elements $x_i(\theta)\in X(\theta)\setminus X^c$. For all $x_i(\theta)\in X(\theta)\setminus X^c$ we have $\mu=1$ if $\theta=\overline{\theta}$ and $\mu=0$ if $\theta=\underline{\theta}$. Hence, $\overline{u}_2(\overline{x}_i)=\overline{u}_2(\overline{x}_j)$ for all $\overline{x}_i,\overline{x}_j\in\overline{X}\setminus X^c$ and $\underline{u}_2(\underline{x}_i)=\underline{u}_2(\underline{x}_j)$ for all $\underline{x}_i,\underline{x}_j\in\underline{X}\setminus X^c$. Consider a weighted average of outcomes in $X(\theta)\setminus X^c$:

$$x^a(\theta)=\frac{\sum\limits_{i:x_i\in X(\theta)\setminus X^c}\pi_i(\theta)x_i}{\sum\limits_{i:x_i\in X(\theta)\setminus X^c}\pi_i(\theta)}.$$

Reduce the support of the original lottery by replacing the elements from $X(\theta)\setminus X^c$ by a single element $x^a(\theta)$ that is assigned with probability $\pi^a(\theta)=\sum\limits_{i:x_i\in X(\theta)\setminus X^c}\pi_i(\theta)$. It does not affect the incentive constraints $IC_1(\theta)$ and the participation constraints $PC_1(\theta)$. Moreover, it increases the expected payoff of P$_1$ due to the concavity of v_1. That is, $v_1(x^a(\theta)) > \sum_{i:x_i\in X(\theta)\setminus X^c} \pi_i(\theta)v_1(x_i(\theta))$. Thus, P$_1$ gains by reducing the number of non-common elements of the support to one element.

Claim 3. *Suppose that $\#(X^c)\geq 2$. Then P_1 is better off by offering a lottery with $\#(X^c)=1$.*

Proof. When the outcome of the contract belongs to the support X^c, each allocation $x_k\in X^c$ from the support induces the belief

$$\mu_k = \frac{\nu\overline{\pi}_k}{\nu\overline{\pi}_k+(1-\nu)\underline{\pi}_k}.$$

The rent obtained by the agent equal to $\Delta\theta\underline{q}_k = \underline{\theta} - \Delta\theta\frac{\nu}{1-\nu}\frac{\overline{\pi}_k}{\underline{\pi}_k}$. The incentive constraint of the high valuation agent, $\overline{IC}_1$, writes

$$\begin{aligned} &\overline{\pi}^a(-\overline{t}^a+\overline{\theta}\overline{q}^a)+\sum_{X^c}\overline{\pi}_k(-t_k+\overline{\theta}q_k+\Delta\theta(\underline{\theta}-\Delta\theta\frac{\nu}{1-\nu}\frac{\overline{\pi}_k}{\underline{\pi}_k}))\\ \geq\ &\underline{\pi}^a(-\underline{t}^a+\overline{\theta}\underline{q}^a)+\sum_{X^c}\underline{\pi}_k(-t_k+\overline{\theta}q_k+\Delta\theta(\underline{\theta}-\Delta\theta\frac{\nu}{1-\nu}\frac{\overline{\pi}_k}{\underline{\pi}_k})).\end{aligned}$$

Let us replace this lottery with the one that has a single common element x^c which is assigned to type θ with probability $\pi(\theta)=\sum_k \pi_k(\theta)$. The belief induced by this lottery is

$$\mu^c = \frac{\nu\overline{\pi}^c}{\nu\overline{\pi}^c+(1-\nu)\underline{\pi}^c}.$$

Reducing the spread of the lottery increases the payoff of P$_1$ due to the concavity of the payoff. To prove that this lottery relaxes the incentive constraint of $\overline{\theta}-$ agent, we have to show that

$$\sum_k\overline{\pi}_k\frac{\overline{\pi}_k}{\underline{\pi}_k}\geq\sum_k\overline{\pi}_k\cdot\frac{\sum_k\overline{\pi}_k}{\sum_k\underline{\pi}_k}.$$

Denote $\pi_1\pi_{\{i\}}\pi_n$ the product of all $\pi_j, j=1,...,n$ except for π_i, and $\#(X^c)$=n. Then,

$$\begin{aligned}\sum\underline{\pi}_k\sum\overline{\pi}_k\frac{\overline{\pi}_k}{\underline{\pi}_k}\ &\geq\ \sum\overline{\pi}_k\sum\overline{\pi}_k\\ &\Leftrightarrow\ \sum\underline{\pi}_k\frac{\sum\overline{\pi}_k^2[\underline{\pi}_1\underline{\pi}_{\{k\}}\underline{\pi}_n]}{\prod\underline{\pi}_k}\geq\sum\overline{\pi}_k\sum\overline{\pi}_k\\ &\Leftrightarrow\ \frac{\sum\underline{\pi}_k}{\prod\underline{\pi}_k}[\sum\overline{\pi}_k^2[\underline{\pi}_1\underline{\pi}_{\{k\}}\underline{\pi}_n]-\frac{\prod\underline{\pi}_k}{\sum\underline{\pi}_k}\sum\overline{\pi}_k\sum\overline{\pi}_k]\geq 0\\ &\Leftrightarrow\ \frac{1}{\prod\underline{\pi}_k}[\sum\underline{\pi}_k\sum\overline{\pi}_k^2[\underline{\pi}_1\underline{\pi}_{\{k\}}\underline{\pi}_n]-\prod\underline{\pi}_k\sum\overline{\pi}_k\sum\overline{\pi}_k]\geq 0\\ &\Leftrightarrow\ \frac{1}{\prod\underline{\pi}_k}\sum\underline{\pi}_1\underline{\pi}_{\{k\}}\underline{\pi}_{\{j\}}\underline{\pi}_n[\overline{\pi}_k\underline{\pi}_j-\overline{\pi}_j\underline{\pi}_k]^2\geq 0,\quad Q.E.D.\end{aligned}$$

Claim 4. *P_1 does not benefit from offering a non-degenerate lottery to $\underline{\theta}-$agent.*

Proof. The result follows directly from the incentive constraint of the $\underline{\theta}-$ agent.

From Claims 1 to 4 we obtain the result of the lemma. *Q.E.D.*

Proof of Proposition 2

From Lemma 1, the optimal contract C_1 is composed of two outcomes $(\bar{t}_1, \overline{q}_1)$ and $(\underline{t}_1, \underline{q}_1)$, and a randomization strategy σ. Proposition 3.1 specifies the optimal contract of P_2 following any message θ. To complete the characterization of feasible contracts of P_1, we should specify the out-of-equilibrium contract offered to a low valuation agent who reports $\overline{\theta}$ to P_1 and is assigned an outcome $(\bar{t}_1, \overline{q}_1)$. Define this contract as

$$\underline{q}_2(\overline{\theta}) = 0 \text{ and } \underline{t}_2(\underline{\theta}) = 0.$$

Then an optimal contract is a solution to the problem:

$$C_1^I: \begin{cases} \max\limits_{(t_1,q_1)} \nu[\sigma(\bar{t}_1 - \frac{1}{2}\overline{q}_1^2) + (1-\sigma)(\underline{t}_1 - \frac{1}{2}\underline{q}_1^2)] + (1-\nu)(\underline{t}_1 - \frac{1}{2}\underline{q}_1^2) \\ \overline{IC}_1: \quad -\bar{t}_1 + \overline{\theta}\overline{q}_1 = -\underline{t}_1 + \overline{\theta}\underline{q}_1 + \Delta\theta \underline{q}_2(\sigma), \\ \underline{IC}_1: \quad -\underline{t}_1 + \underline{\theta}\underline{q}_1 \geq -\bar{t}_1 + \underline{\theta}\overline{q}_1 + \max\{-\overline{\theta}\Delta\theta, 0\}, \\ \overline{PC}_1: \quad -\bar{t}_1 + \overline{\theta}\overline{q}_1 \geq 0, \\ \underline{PC}_1: \quad -\underline{t}_1 + \underline{\theta}\underline{q}_1 \geq 0, \end{cases}$$

where

$$q_2(\sigma) = \underline{\theta} - \frac{\nu(1-\sigma)}{1-\nu}\Delta\theta.$$

Note that $\overline{IC}_1$ and $\underline{PC}_1$ imply $\overline{PC}_1$. Let us ignore the constraint $\underline{IC}_1$ for the moment, and consider a relaxed problem under constraints $\overline{IC}_1$ and $\underline{PC}_1$. $\underline{PC}_1$ is binding as the payoff of P_1 is increasing in $\underline{t}_1$. The objective of P_1 in the relaxed problem writes:

$$\max_{(\sigma,q_1)} \nu\sigma(\overline{\theta}\overline{q}_1 - \frac{1}{2}\overline{q}_1^2 - \Delta\theta(\underline{q}_1 + \underline{q}_2(\sigma))) + (1-\nu\sigma)(\underline{\theta}\underline{q}_1 - \frac{1}{2}\underline{q}_1^2).$$

The first order conditions of this problem with respect to $\overline{q}_1$ and $\underline{q}_1$ write:

$$\begin{array}{ll} \overline{q}_1: & \overline{q}_1 = \overline{\theta}, \\ \underline{q}_1: & \underline{q}_1 = \underline{\theta} - \Delta\theta\frac{\nu\sigma}{1-\nu\sigma}, \\ \sigma: & [\overline{\theta}\overline{q}_1 - \frac{1}{2}\overline{q}_1^2 - \Delta\theta\underline{q}_1 - \Delta\theta\underline{q}_2(\sigma)] - [\underline{\theta}\underline{q}_1 - \frac{1}{2}\underline{q}_1^2] = \sigma\Delta\theta\frac{d\underline{q}_2(\sigma)}{d\sigma}. \end{array}$$

This solution satisfies $\underline{IC}_1$:

$$0 \geq -\bar{t}_1 + \overline{\theta}\overline{q}_1 - \Delta\theta\overline{q}_1 \Leftrightarrow 0 \geq \Delta\theta(\underline{q}_1 - \overline{q}_1)$$

because $\underline{q}_1 < \overline{q}_1$. Hence, the solution of the relaxed problem is also a solution of C_1^I.

Now consider the conditions that determine the optimal randomization strategy σ. The expected payoff V_1 of P_1 is not concave with respect to σ. However, since V_1 is continuous and differentiable for all $\sigma \in [0,1]$, even if the interior solution $0 < \sigma < 1$ is not unique, it satisfies the following first order condition:

$$\frac{\partial V_1}{\partial \sigma} \equiv [\overline{\theta}\overline{q}_1 - \frac{1}{2}\overline{q}_1^2 - \Delta\theta \underline{q}_1 - \Delta\theta \underline{q}_2(\sigma)] - [\underline{\theta}\underline{q}_1 - \frac{1}{2}\underline{q}_1^2] - \sigma\Delta\theta \frac{d\underline{q}_2(\sigma)}{d\sigma} = 0 \tag{5}$$

and the second order condition:

$$\left.\frac{\partial^2 V_1}{\partial \sigma^2}\right|_{\sigma=\sigma_0} \leq 0 \text{ for all } \sigma_0 \in (0,1) \text{ and satisfy (5).}$$

Replacing the optimal values of $\overline{q}_1$ and $\underline{q}_1$, the first order condition writes

$$\frac{\partial V_1}{\partial \sigma} = \frac{1}{2}\Delta\theta(\Delta\theta - 2\underline{\theta}) - \frac{\nu}{1-\nu}\frac{\nu\sigma(1-2\sigma-\nu\sigma^2)}{(1-\nu\sigma)^2}(\Delta\theta)^2 = 0. \tag{6}$$

So the optimal randomization strategy is a solution of the polynomial of the third degree. Thus, there exists at least one real solution. If the roots of (6) do not belong to $[0,1]$, then the payoff of P_1 is either increasing or decreasing in σ, and the optimal σ is either 1 or 0, respectively. The necessary conditions to have corner solutions $\sigma = 0$ or $\sigma = 1$ are

$$\begin{aligned} \left.\frac{\partial V_1}{\partial \sigma}\right|_{\sigma=0} &= \frac{1}{2}\Delta\theta(\Delta\theta - 2\underline{\theta}) < 0, \\ \left.\frac{\partial V_1}{\partial \sigma}\right|_{\sigma=1} &= \frac{1}{2}\Delta\theta(\Delta\theta - 2\underline{\theta}) + \frac{\nu^2(1+\nu)}{(1-\nu)^2}(\Delta\theta)^2 > 0. \end{aligned}$$

If at least some roots of (6) do belong to [0,1] and satisfy the second order condition, then the optimal σ is either the root of (6), or the corner solution. *Q.E.D.*

Proof of Proposition 3.

The expected payoff of P_1 and P_2 are, respectively:

$$\begin{aligned} V_1^I &= \nu\sigma(\frac{1}{2}\overline{\theta}^2 - \Delta\theta(2\underline{\theta} - \frac{\nu\sigma}{1-\nu\sigma}\Delta\theta - \frac{\nu(1-\sigma)}{1-\nu}\Delta\theta)) \\ &\quad +(1-\nu\sigma)(\frac{1}{2}\underline{\theta}^2 - \frac{1}{2}\left(\frac{\nu\sigma}{1-\nu\sigma}\right)^2(\Delta\theta)^2), \\ V_2^I &= \nu\sigma\frac{1}{2}\overline{\theta}^2 + \nu(1-\sigma)(\frac{1}{2}\overline{\theta}^2 - \Delta\theta(\underline{\theta} - \frac{\nu(1-\sigma)}{1-\nu}\Delta\theta)) \\ &\quad +(1-\nu)(\frac{1}{2}\underline{\theta}^2 - \frac{1}{2}\left(\frac{\nu(1-\sigma)}{1-\nu}\right)^2(\Delta\theta)^2). \end{aligned}$$

Therefore,

$$V_1^I - V_2^I = -\Delta\theta\underline{\theta}\nu(1+\sigma) + (\Delta\theta)^2\frac{\nu}{2}(-\frac{\nu(3\sigma^2+1)}{1-\nu} + \frac{\nu\sigma^2 - 2(1-\nu\sigma^2) - 2\sigma(1-\nu)}{1-\nu\sigma}).$$

The sign of the second part of the expression is the same as the sign of

$$\begin{aligned} & (1-\nu)(\nu\sigma^2 - 2(1-\nu\sigma^2) - 2\sigma(1-\nu)) - (1-\nu\sigma)\nu(3\sigma^2+1) \\ = & -3\nu^2\sigma^2(1-\sigma) - 2\sigma(1-\nu) - (2 - 2\nu\sigma + \nu^2\sigma - \nu). \end{aligned}$$

To verify that $2 - 2\nu\sigma + \nu^2\sigma - \nu > 0$, note that the function $F(\sigma) = 2 - 2\nu\sigma - \nu^2\sigma - \nu$ is decreasing in σ, and $F(1) = 2 - 3\nu + \nu^2 > 0$ for all $\nu \in [0, 1]$. Hence, $V_1^I \leq V_2^I$. *Q.E.D.*

Proof of Proposition 6.

The difference between the expected payoffs of P_1 under the postponed and immediate implementation schedules is equal to

$$\begin{aligned} V_1^P - V_1^I &= \nu(\tfrac{1}{2}\overline{\theta}^2 - \Delta\theta(\underline{\theta} - \frac{\nu}{1-\nu}\Delta\theta)) + (1-\nu)(\tfrac{1}{2}\underline{\theta}^2 - \tfrac{1}{2}(\Delta\theta)^2(\frac{\nu}{1-\nu})^2) \\ &\quad -\nu\sigma(\tfrac{1}{2}\overline{\theta}^2 - \Delta\theta(2\underline{\theta} - \frac{\nu(1-\sigma)}{1-\nu}\Delta\theta - \frac{\nu\sigma}{1-\nu\sigma}\Delta\theta)) - (1-\nu\sigma)(\tfrac{1}{2}\underline{\theta}^2 - \tfrac{1}{2}(\Delta\theta)^2(\frac{\nu\sigma}{1-\nu\sigma})^2) \\ &= \nu\sigma\underline{\theta}\Delta\theta + \tfrac{1}{2}\nu(1-\sigma)(\Delta\theta)^2 + \tfrac{1}{2}\nu^2\Delta\theta(\frac{1-2\sigma(1-\sigma)}{1-\nu} - \frac{\sigma^2}{1-\nu\sigma}). \end{aligned}$$

Note that $1-2\sigma(1-\sigma)-\sigma^2 = (1-\sigma)^2 \geq 0$ and $1-\nu < 1-\nu\sigma$ for $\sigma < 1$. Thus, $(\frac{1-2\sigma(1-\sigma)}{1-\nu} - \frac{\sigma^2}{1-\nu\sigma}) \geq 0$. Therefore, $V_1^P > V_1^I$.

The benefit of postponing the implementation increases in $\Delta\theta$:

$$\frac{d(V_1^P - V_1^I)}{d(\Delta\theta)} > 0.$$

and increases in ν:

$$\begin{aligned} \frac{d(V_1^P - V_1^I)}{d\nu} &= \sigma\underline{\theta}\Delta\theta + \tfrac{1}{2}(1-\sigma)(\Delta\theta)^2 + \nu\Delta\theta(\frac{1-2\sigma(1-\sigma)}{1-\nu} - \frac{\sigma^2}{1-\nu\sigma}) \\ &\quad + \tfrac{1}{2}\nu^2\Delta\theta(\frac{1-2\sigma(1-\sigma)}{(1-\nu)^2} - \frac{\sigma^3}{(1-\nu\sigma)^2}). \end{aligned}$$

To see that $\frac{d(V_1^P - V_1^I)}{d\nu} > 0$, note also that $1 - 2\sigma(1-\sigma) - \sigma^3 = (1-\sigma)^2 + \sigma^2(1-\sigma) \geq 0$. *Q.E.D.*

Proof of Proposition 7.

The difference between the payoffs of the agent under postponed and immediate implementation schedules is equal to

$$\overline{u}^P - \overline{u}^I = \nu(\Delta\theta)^2(\frac{1+\sigma}{1-\nu} - \frac{\sigma}{1-\nu\sigma}).$$

It is positive as $1 + \sigma > \sigma$ and $1 - \nu < 1 - \nu\sigma$. *Q.E.D.*

Proof of Proposition 8.

The difference between the payoffs of P_2 under the two contracting schedules is equal to

$$V_2^P - V_2^I = -\nu\sigma\Delta\theta(\underline{\theta} - \frac{1}{2}\frac{1-(1-\sigma)^2}{\sigma}\frac{\nu}{1-\nu}\Delta\theta).$$

It is negative. To see why, recall first that the output of the low valuation agent is positive under the prior beliefs, $\underline{\theta} - \frac{\nu}{1-\nu}\Delta\theta > 0$. Then $\underline{\theta} - \frac{1}{2}\frac{1-(1-\sigma)^2}{\sigma}\frac{\nu}{1-\nu}\Delta\theta > 0$ if $\frac{1}{2}\frac{1-(1-\sigma)^2}{\sigma} \leq 1$. The last condition is equivalent to $\sigma^2 \geq 0$, which is satisfied for any σ. *Q.E.D.*

Proof of Proposition 9.

The derivation of the optimal contract C_2^R is analogous to C_1^I, so we omit it. To verify that $\frac{d\sigma}{d\overline{u}_1^0} \geq 0$, not that $\frac{d\sigma}{d\overline{u}_1^0} = -\frac{d^2V_1/d\overline{u}_1^0 d\sigma}{d^2V_1/d\sigma^2} = -\frac{1}{d^2V_1/d\sigma^2} \geq 0$. The last inequality holds for any interior solution $\sigma \in (0,1)$ due to the second order condition $\frac{d^2V_1}{d\sigma^2} \leq 0$. *Q.E.D.*

References

[1] Admati, A. and M. Perry, 1987, ”Strategic Delay in Bargaining”, *Review of Economic Studies*, LIV: 345-364.

[2] Benveniste, L., W. Busada and W. Wilhelm, 2002, "Information Externalities and the Role of Underwriters in Primary Equity Markets", *Journal of Financial Intermediation*, 11: 61-86.

[3] Chamley, C. and D. Gale, 1994, ”Information Revelation and Strategic Delay in a Model of Investment”, *Econometrica*, 65(2): 1065-1085.

[4] Gibbard, A, 1977, ”Manipulation of Voting Schemes”, *Econometrica*, 41: 587-601.

[5] Green, J. and J.-J. Laffont, 1977, ”Characterization of Satisfactory mechanisms for the Revelation of Preferences for Public Goods”, *Econometrica*, 45: 427-438.

[6] Gu, W. and P. Kuhn, 1998, ”A Theory of Holdouts in Wage Bargaining”, *American Economic Review*, 88(3): 428-449.

[7] Gu, W. and P. Kuhn, 1999, ”Learning in Sequential Wage Negotiations: Theory and Evidence”, *Journal of Labor Economics*, 17(1): 109-140.

[8] Hendricks, K. and D. Kovenock, 1989, ”Asymmetric Information, Information Externalities, and Efficiency: The Case of Oil Exploration”, *RAND Journal of Economics*, 20(2): 164-182.

[9] Kartasheva, A, 2004, The Revelation Principle under Sequential Contracting, mimeo, University of Toulouse.

[10] Laffont, J.-J. and D. Martimort, 2002, The Theory of Incentives: The Principal-Agent Model, Princeton University Press.

[11] Laffont, J.-J. and J. Tirole, 1990, ”Adverse Selection and Renegotiation in Procurement”, Review of Economic Studies, 57: 597-625.

[12] Martimort, D., 1996, ”Exclusive Dealing, Common Agency, and Multiprincipal Incentive Theory”, *RAND Journal of Economics,* 27(1): 1-31.

[13] Miller, D., 2005, "Invention under Uncertainty and the Threat of Ex-post Entry", working paper, UCSD.

[14] Myerson, R., 1979, "Incentive Compatibility and the Bargaining Problem", *Econometrica*, 47: 61-73.

[15] Pindyck, Robert S., 1991, "Irreversibility, Uncertainty and Investment", *Journal of Economic Literature,* 39: 1110-1148.

[16] Rob, R., 1991, "Learning and Capacity Expansion under Demand Uncertainty", *Review of Economic Studies*, 58(4): 655-675.

[17] Strausz, R., 2006, "Deterministic versus Stochastic Mechanisms in Principal-Agent Models", *Journal of Economic Theory, forthcoming.*